

# Magnetodynamics Inside and Outside Magnetars

Xinyu Li

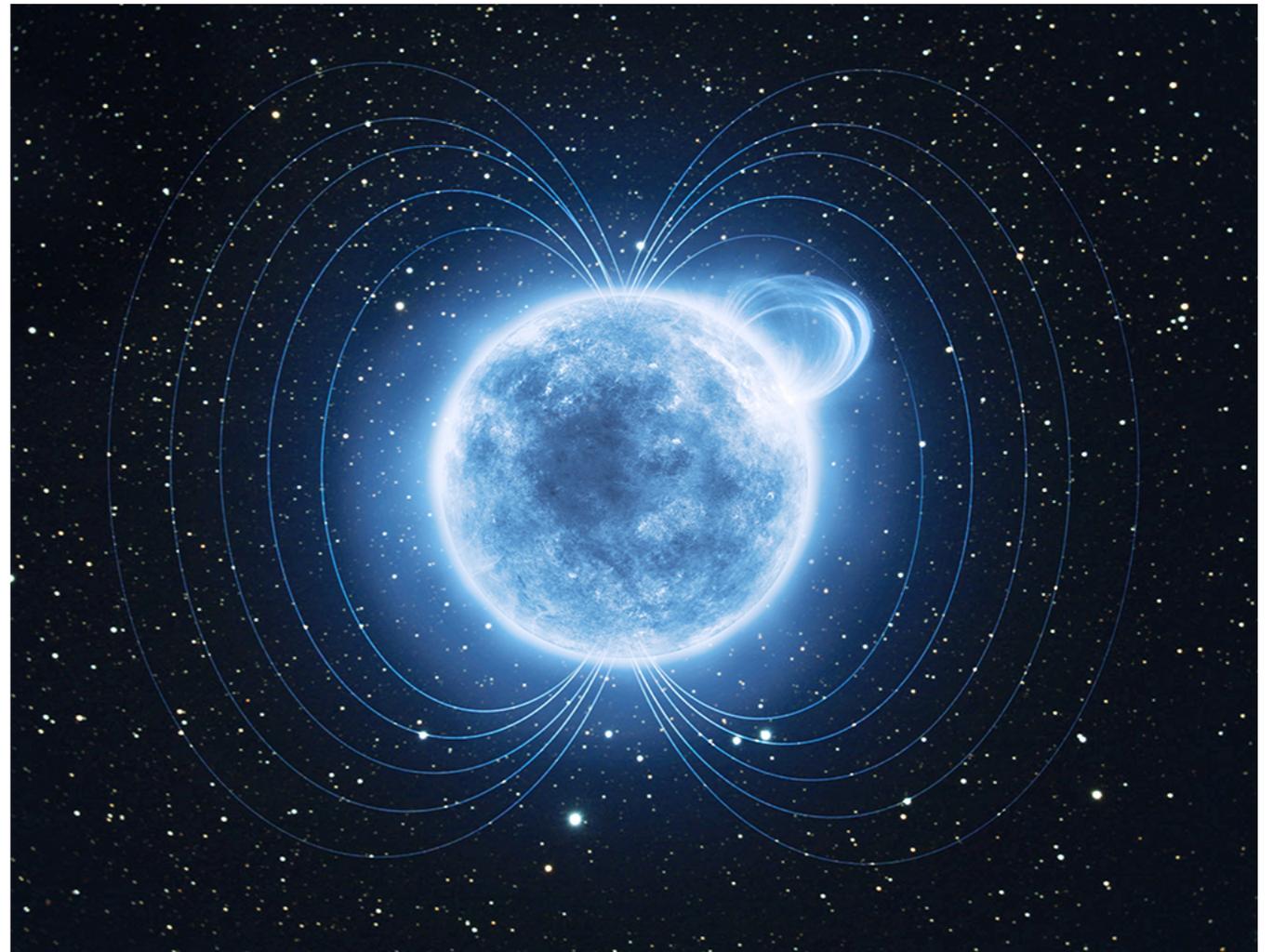
Canadian Institute for Theoretical Astrophysics / Perimeter Institute  
with Andrei Beloborodov, Yuri Levin, Jonathan Zrake, Lorenzo Sironi



# Magnetar

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- Neutron star with ultra-strong magnetic fields
- Slow rotation period 1-10s
- ~30 sources have been identified by now
- McGill Online catalogue list 29 magnetars
- Online interactive analysis:  
[http://  
magnetars.ice.csic.es](http://magnetars.ice.csic.es)

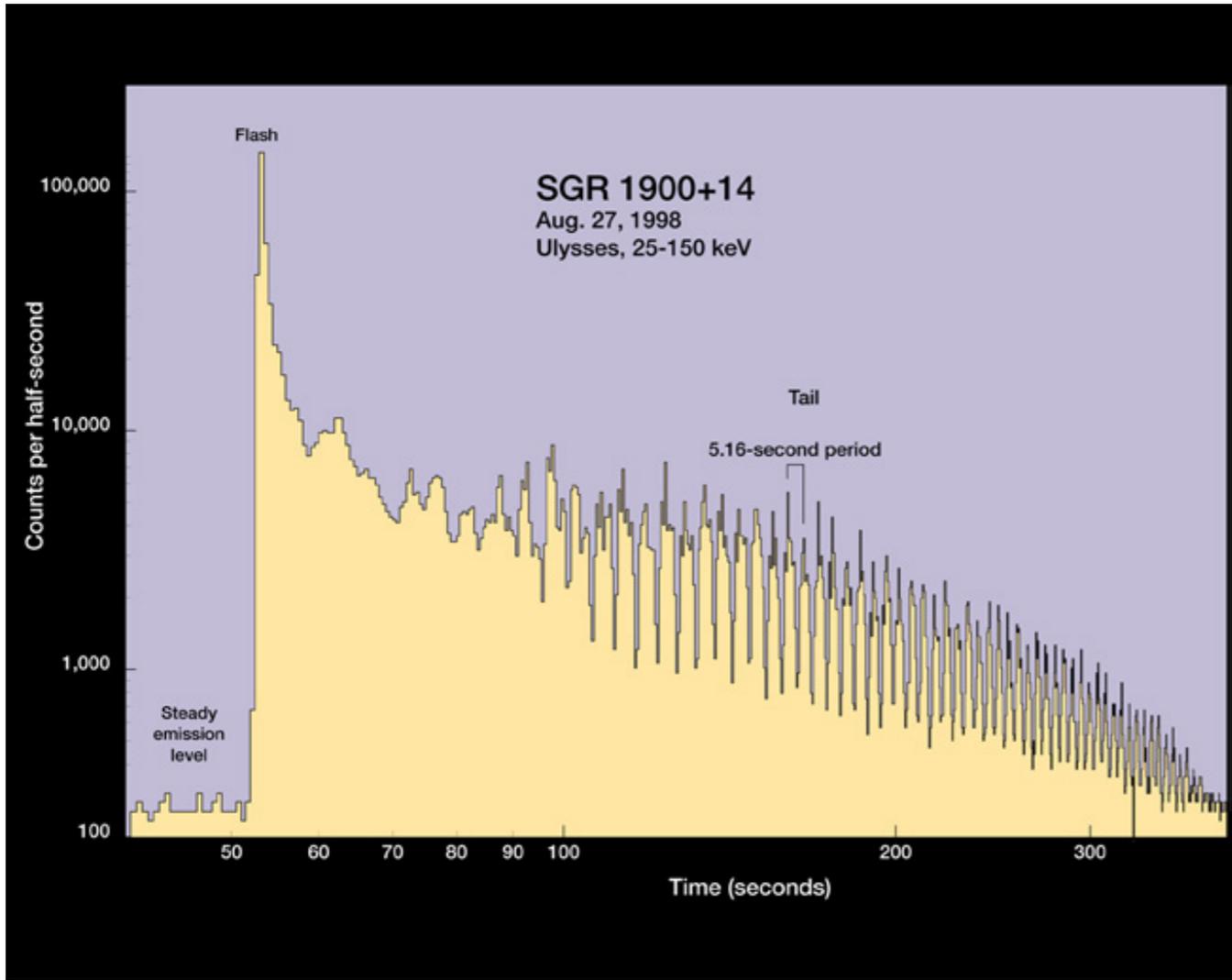


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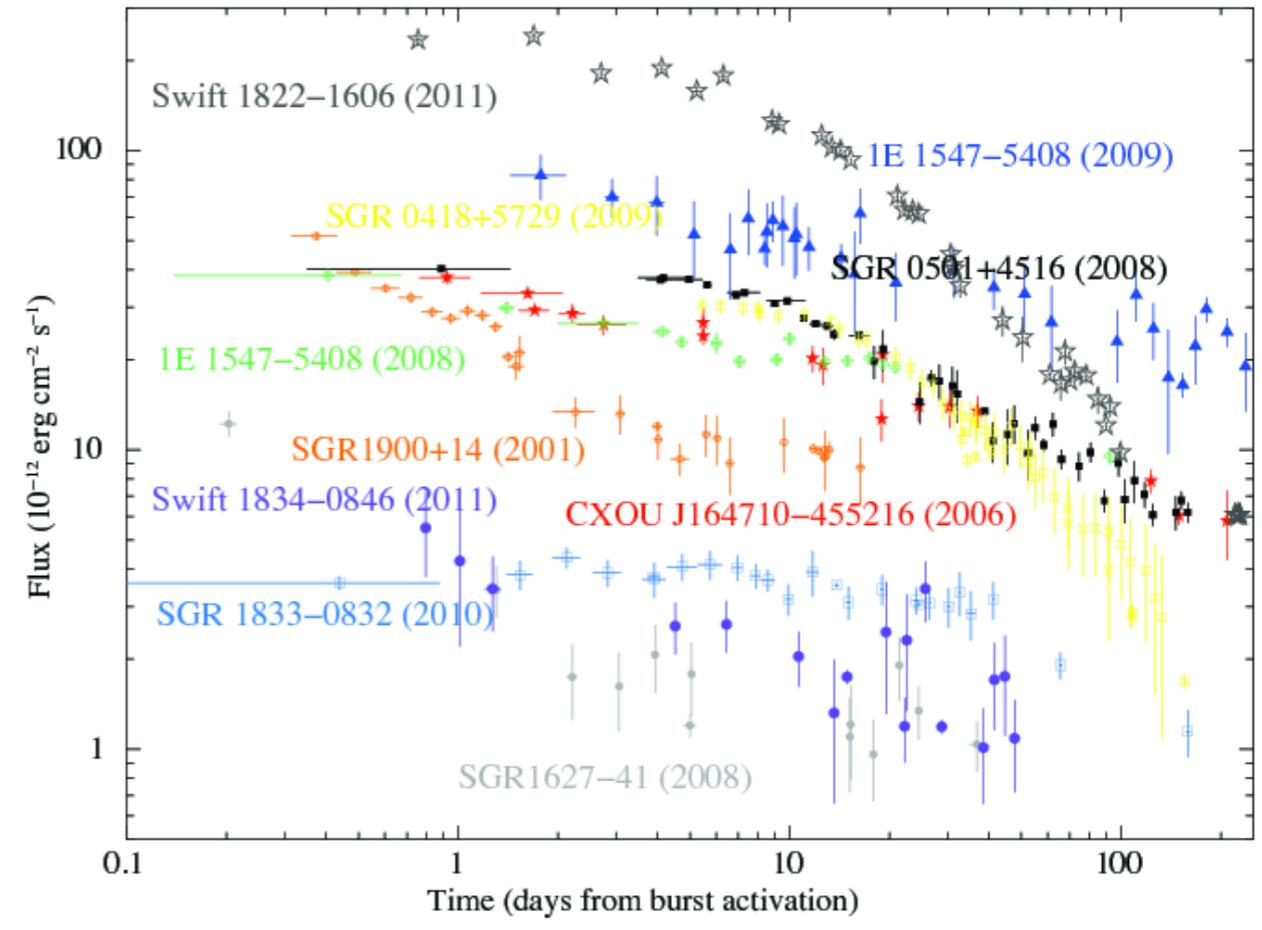
# Magnetar Activities

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- **Giant flares:** peak luminosity  $10^{43}$ - $10^{47}$  erg/s, rises in milliseconds
- **Soft gamma ray bursts:** less energetic  $\sim 10^{43}$  erg/s
- **Outbursts:** 10-1000 times increase of X-ray luminosity ( $\sim 10^{36}$  erg/s) and decays exponentially for months to years
- **Persistent X-ray emission:** luminosity usually much larger than the spin-down power
- **High surface temperature:**  $10^{35}$  erg/s (arXiv:1605.09077)
- **Timing anomalies:** glitches and anti-glitches (sudden spin-down)



Giant Flare



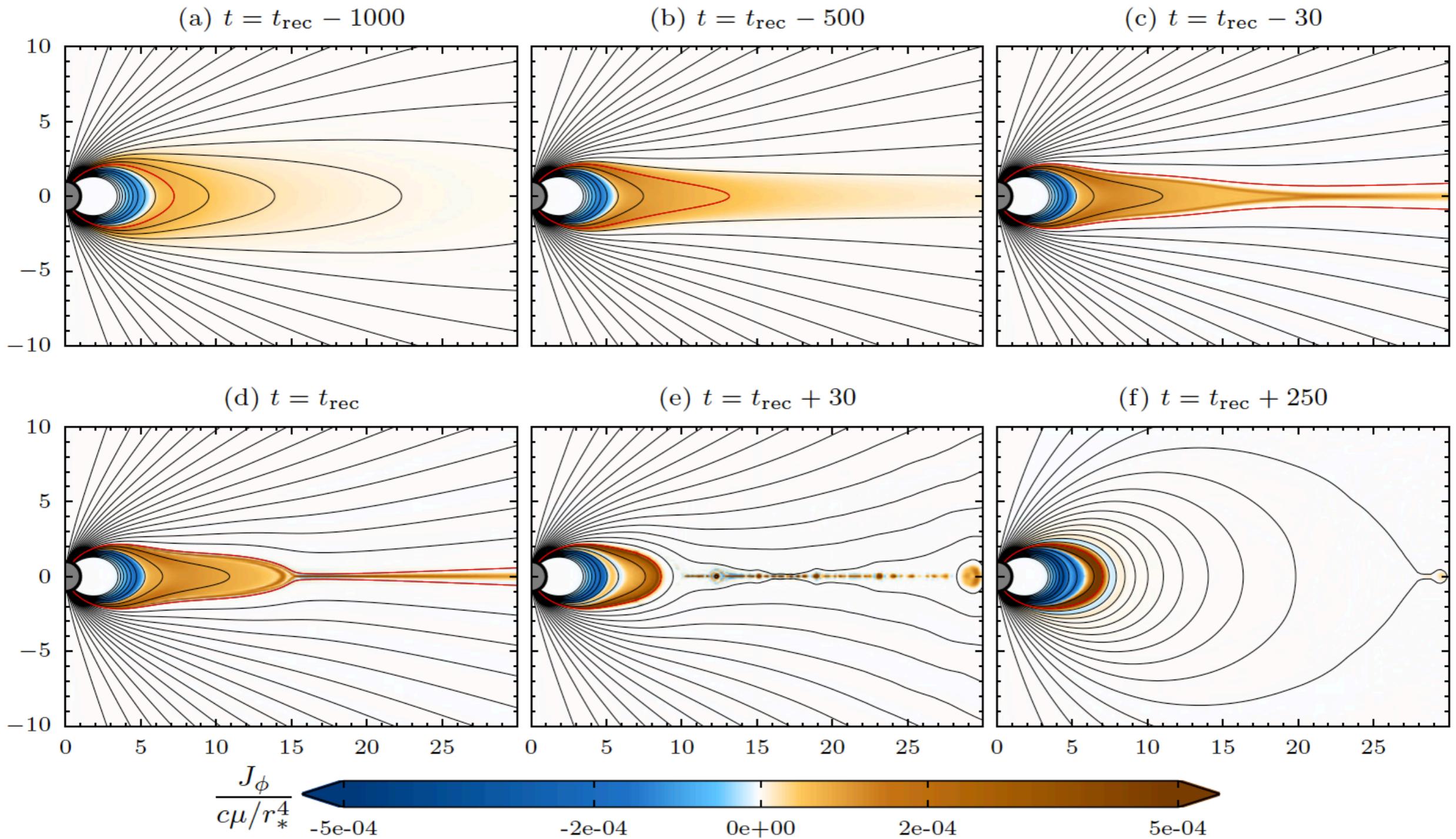
Outbursts

# Key Questions

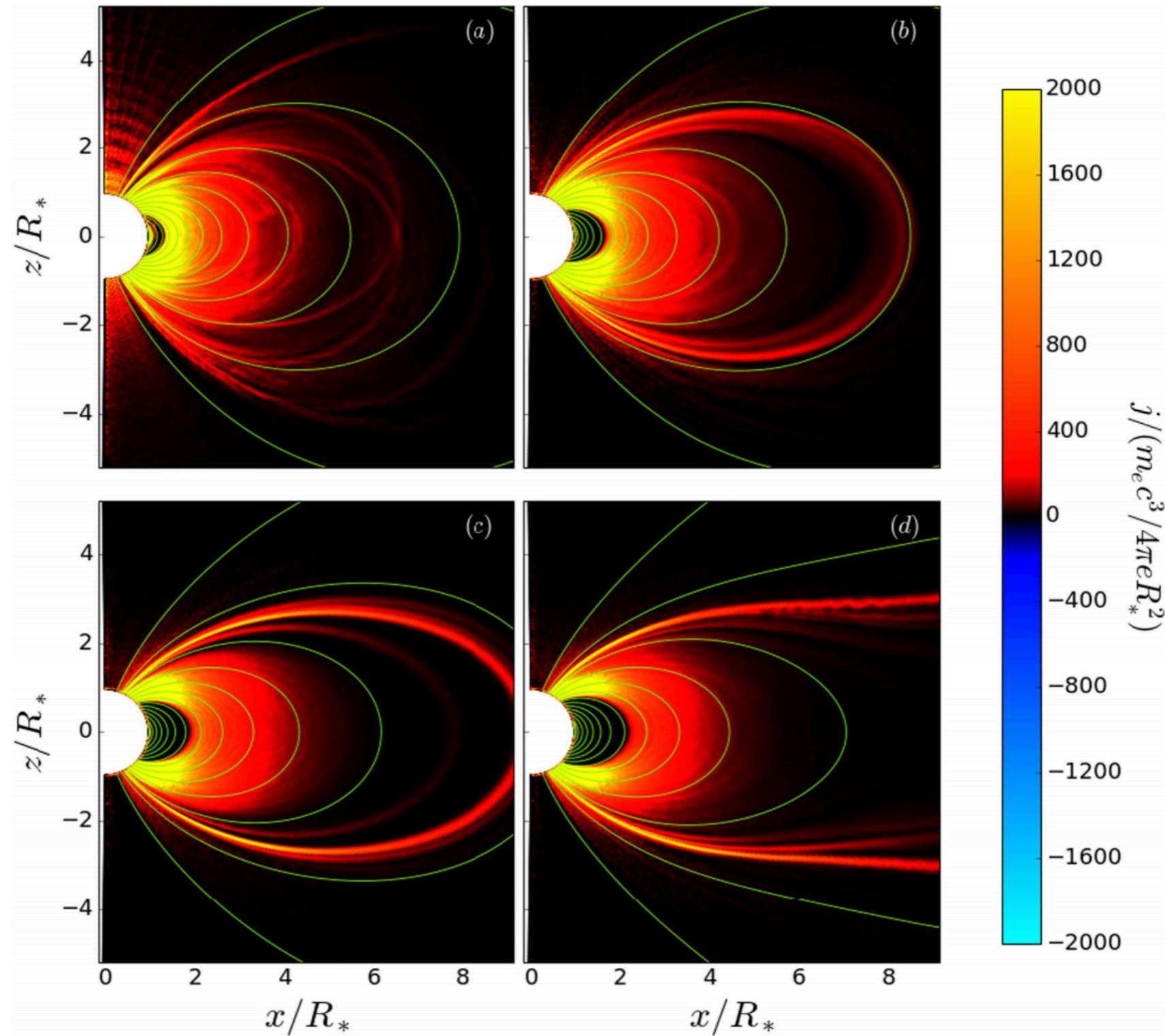
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- Thompson & Duncan (1995, 1996) proposed that bursts and giant flares are powered by the magnetic energy and coined the word “magnetar”.
- **1) (fast) conversion of magnetic energy to radiations**
- Magnetic energy is converted to radiations possibly through a distorted magnetosphere (Parfrey 2013, Beloborodov 2009, 2013).
- **2) origin of magnetospheric distortion**

# Strongly Twisted Magnetosphere (Parfrey 2013)



# Untwisting of Magnetosphere (Chen & Beloborodov 2017)



# Outline

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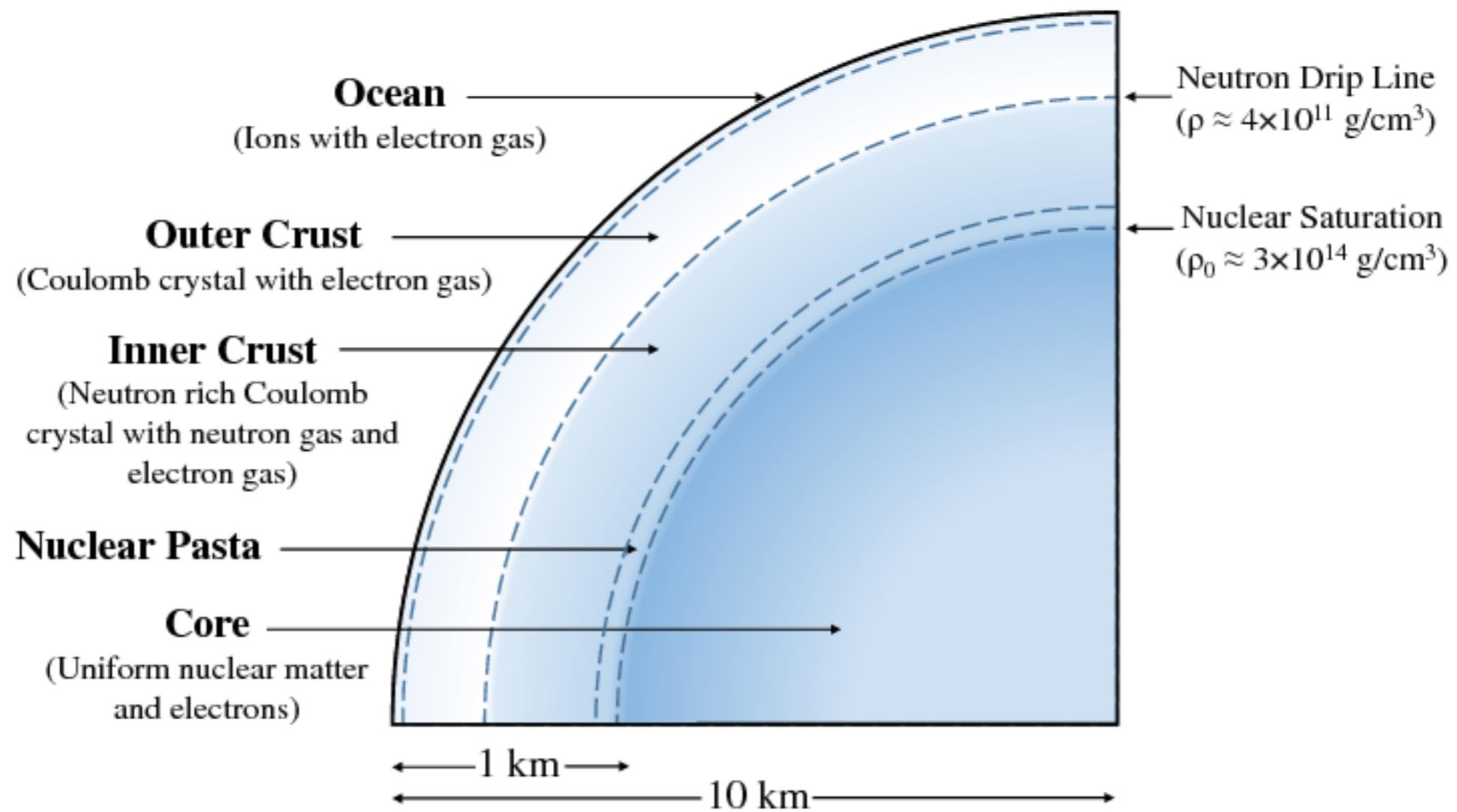
- Magnetodynamics inside the magnetar:
  - 1) Plastic deformation of the magnetar crust by magnetic stress
  - 2) Modeling magnetar outbursts with Hall waves induced plastic failures (1606.04895)
  - 3) Thermal afterglow of giant flares by crustal damping of Alfvén waves (1505.03465 )
- Magnetodynamics outside the magnetar:

Magnetospheric dissipation of Alfvén waves (1810.10493)

Origin of  
Magnetospheric  
Distortion

How Magnetic  
Energy is  
Dissipated

# Neutron Star Crust

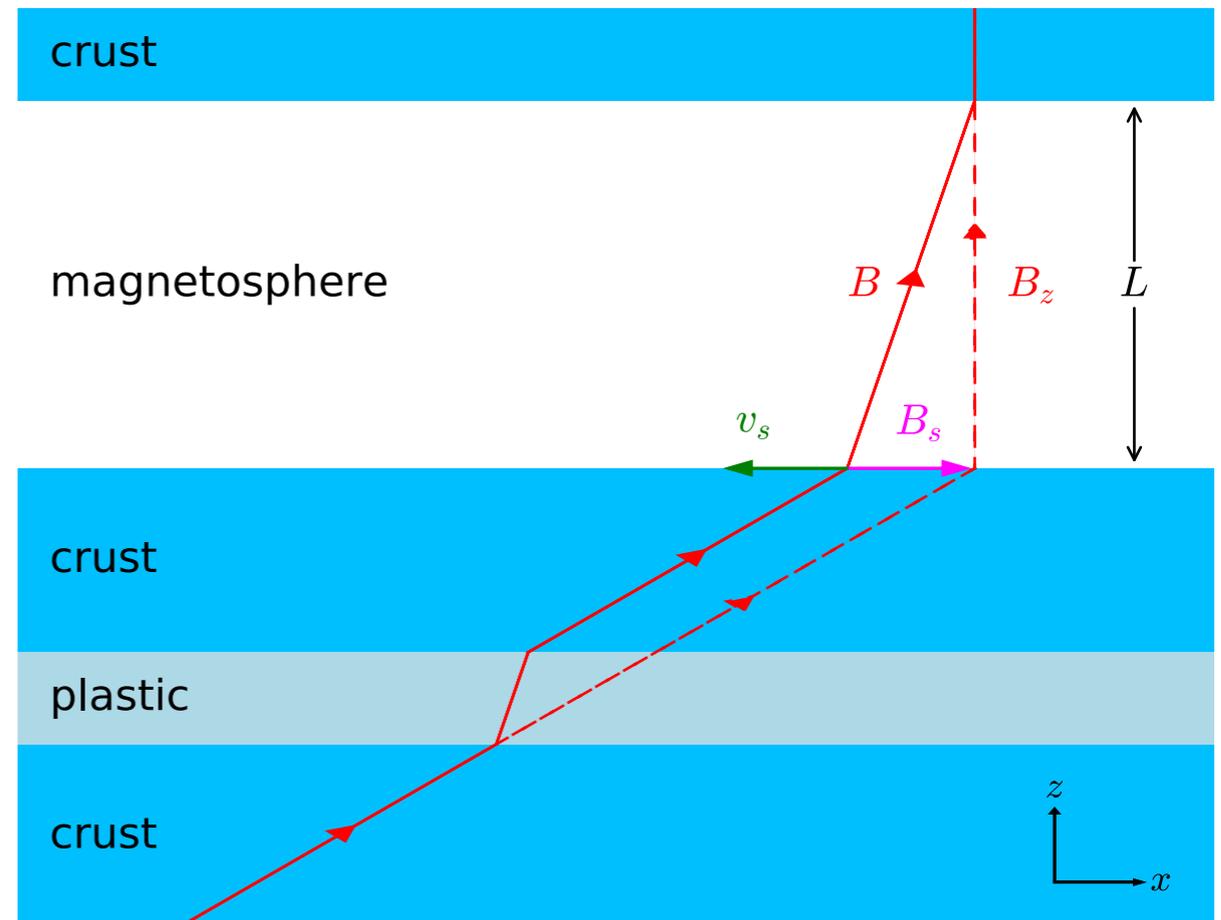
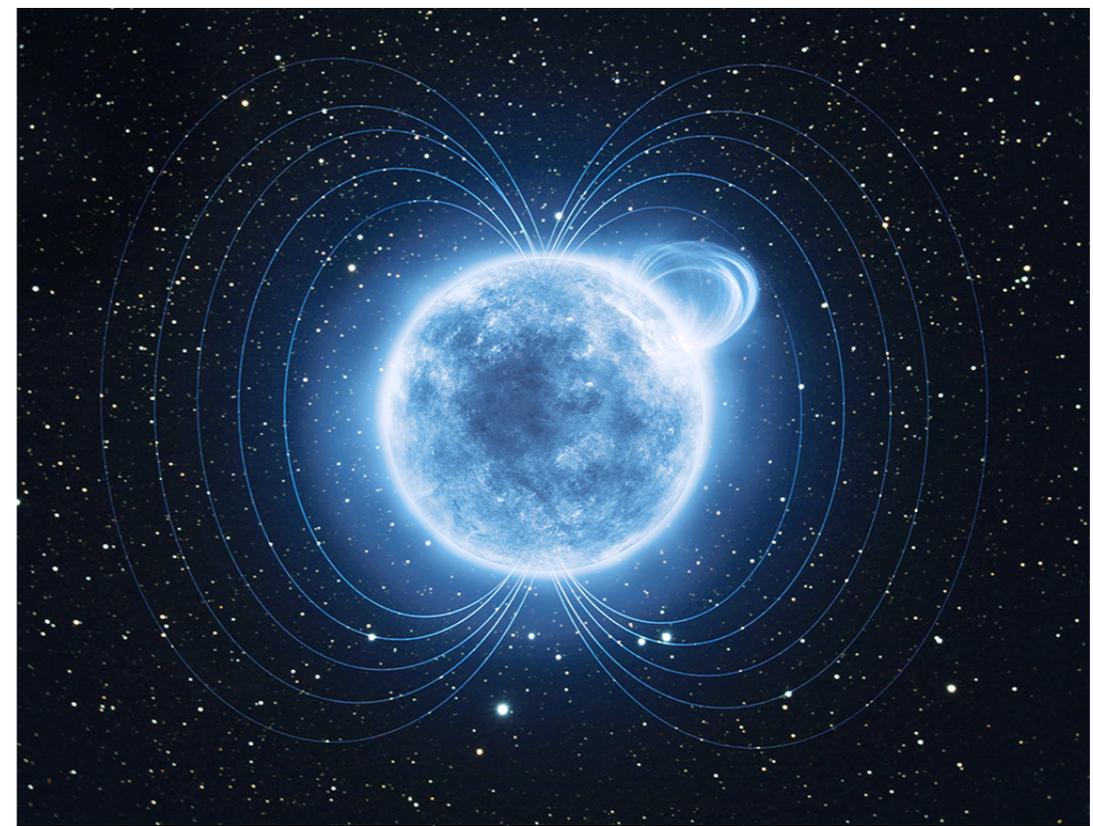


# Plastic Deformation of the Crust

- The strong magnetic stress is able to trigger the plastic failure when the magnetic stress exceeds  $\sigma_{cr} \approx 0.1\mu$

$$B_{cr} = 4\pi\sigma_{cr}/B_z$$

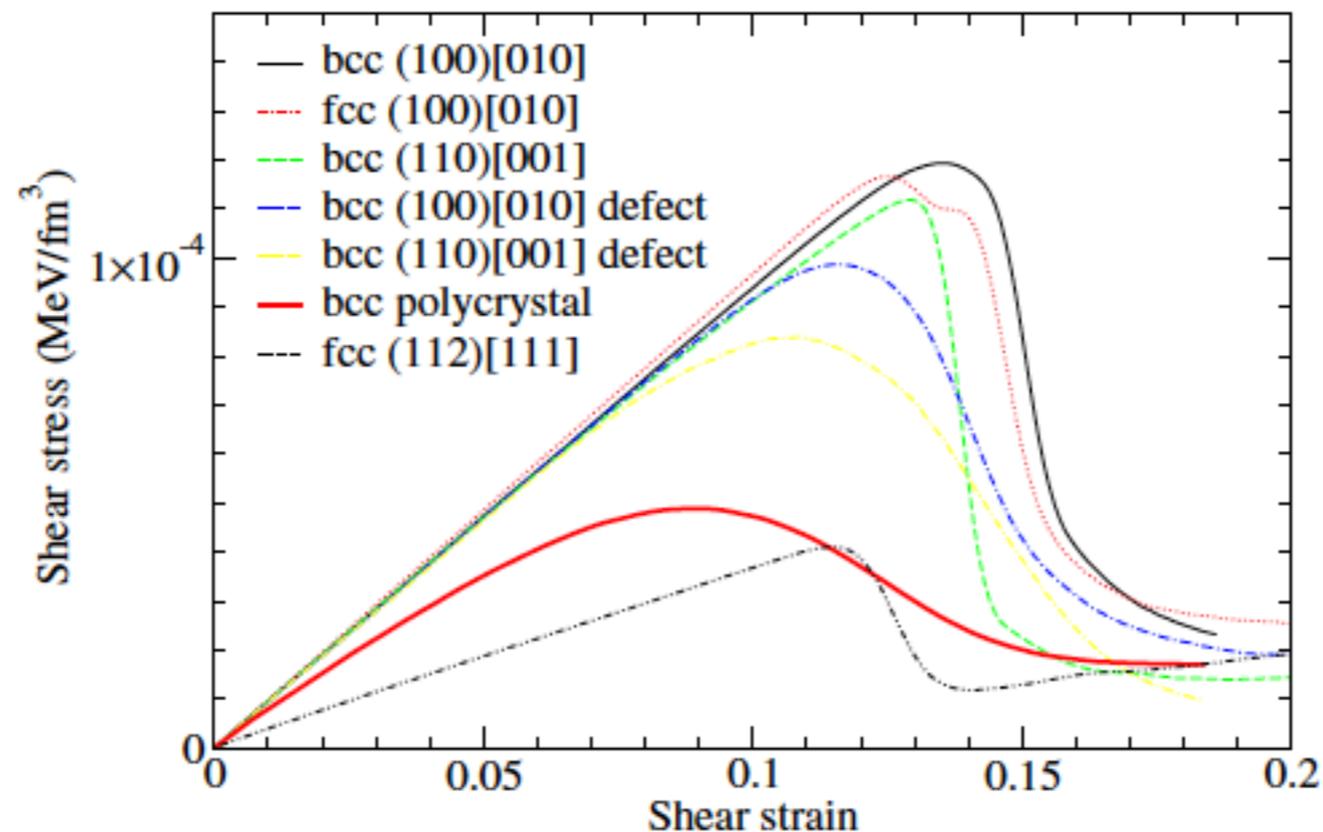
- Magnetic energy is dissipated to heat in the plastic flow
- The magnetic field lines are dragged by the plastic flow and induces footpoint motion which gives rise to the toroidal field



# Thermal Responses of the Crust

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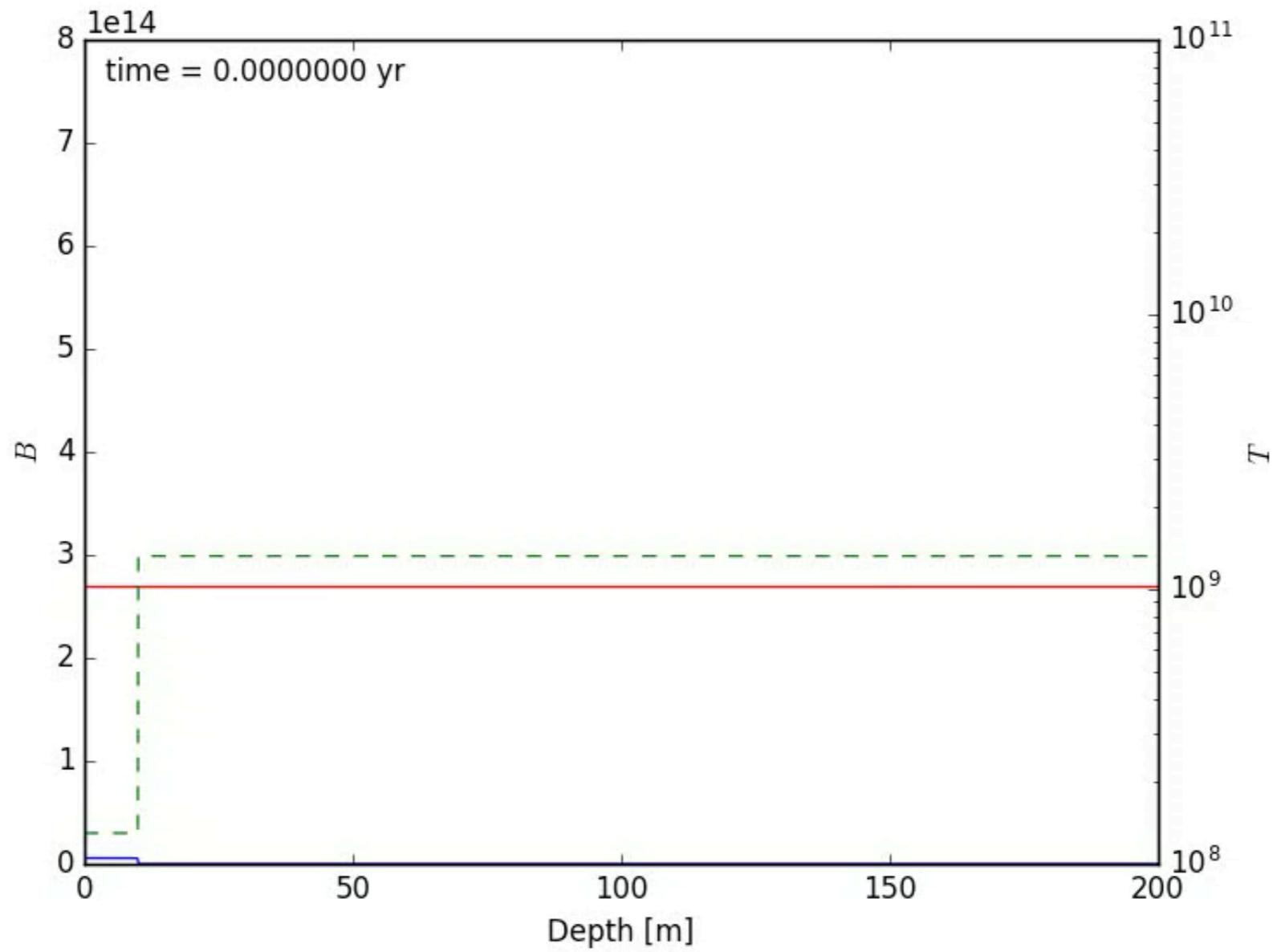
- The crustal material is softened by the increasing temperature (Chugunov & Horowitz 2010)
- There is also a phase transition in the plastic flow (Horowitz & Kadau 2009)



# Thermoplastic Waves (TPW)

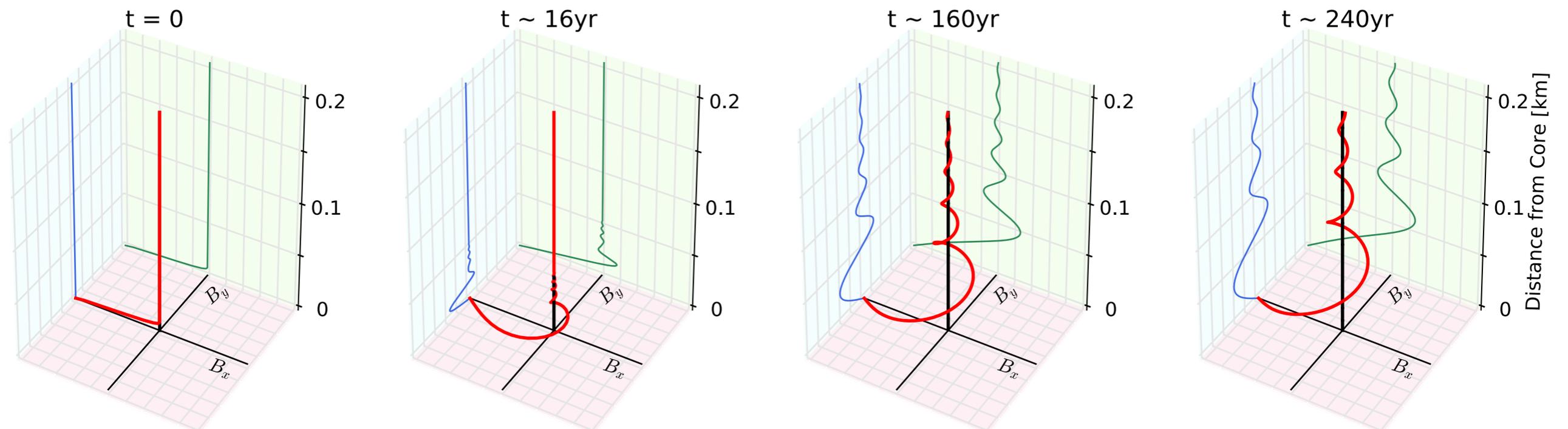
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- Beloborodov & Levin 2014
- The plastic flow dissipates magnetic energy to heat and diffuses it to the neighboring regions
- The neighboring crustal material is softened by the temperature increase and more plastic failures can be triggered
- The speed of TPW is  $v = \sqrt{\frac{\chi\mu B}{\eta}}$



# Hall Waves in the Crust

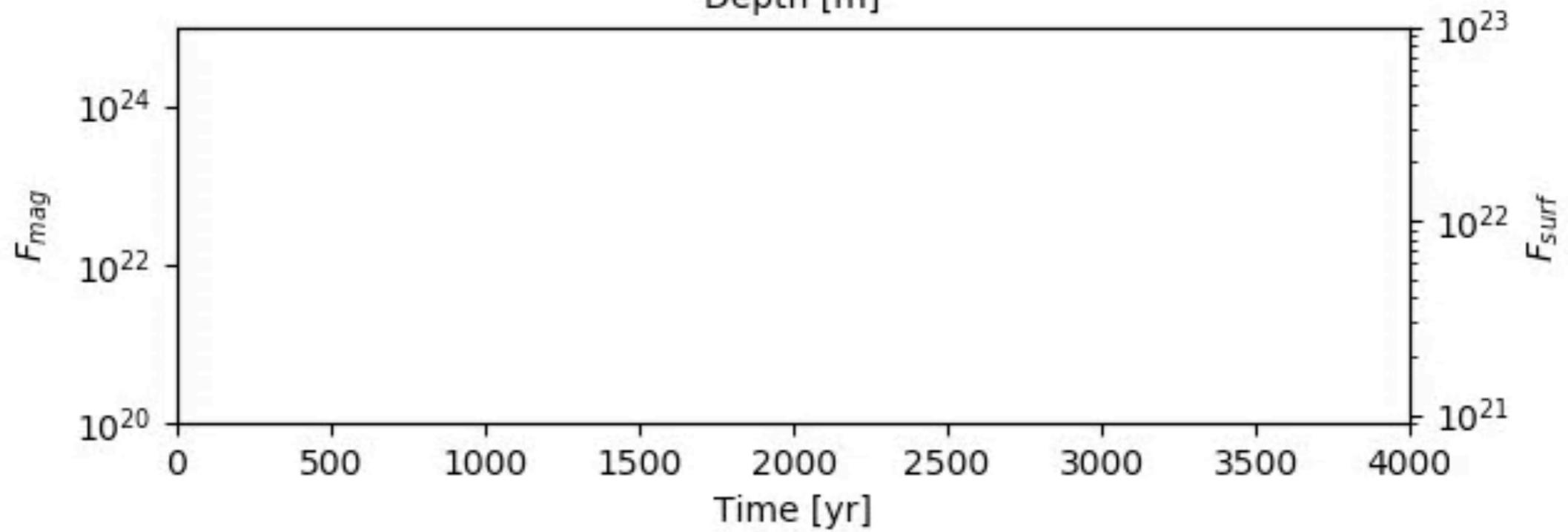
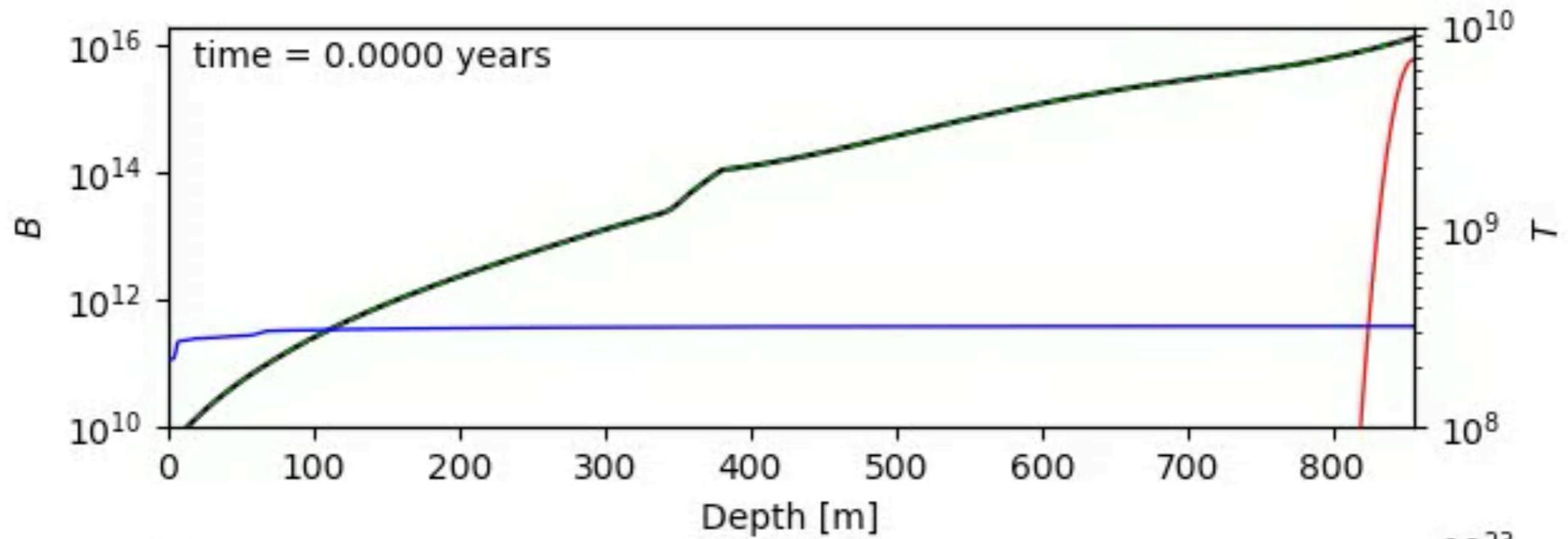
- Hall waves: magnetic field advected by the electron fluid
- Hall waves generated from the core-crust interface accumulate magnetic energy inside the crust.
- Plastic flow is initiated when the magnetic stress is supercritical.

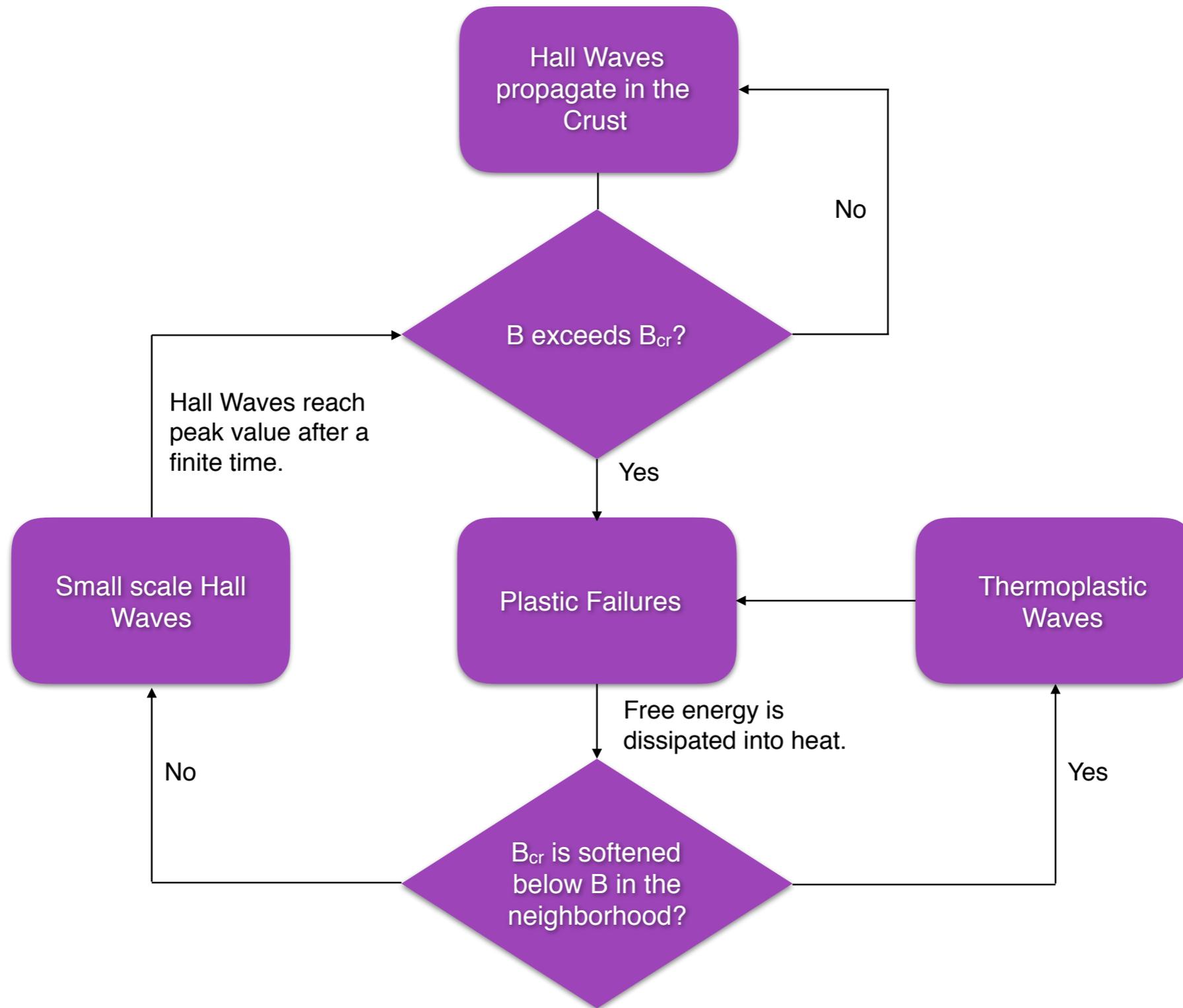


# Crustal Response

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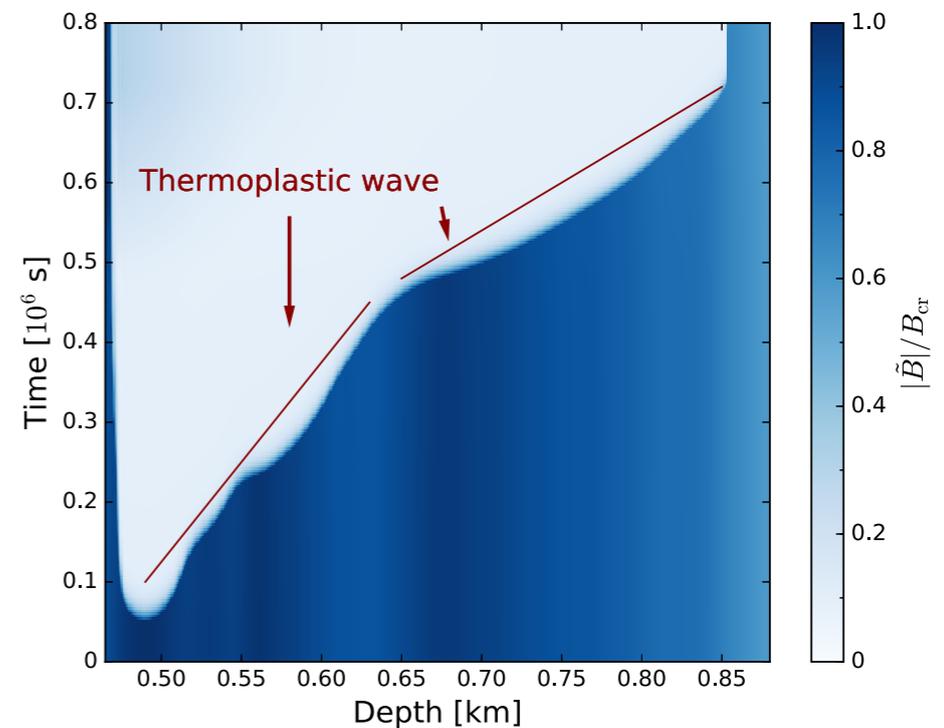
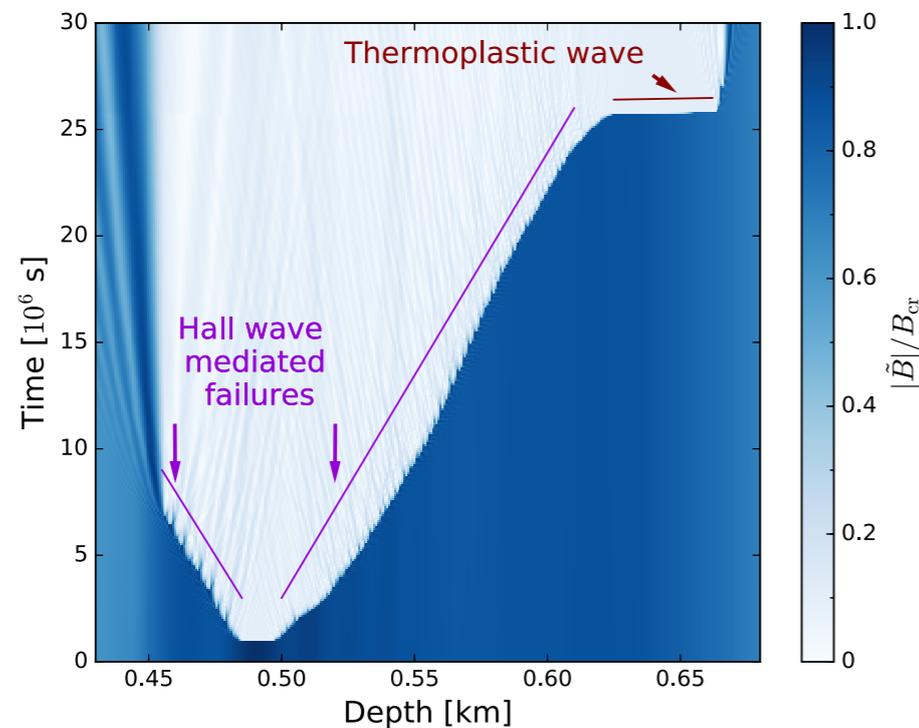
- **Elastic response:** assume stress balance on the slow time scale of Hall wave propagation
- **Plastic response:** shear magnetic is reduced by the plasma viscosity. Set the timescale for the dissipation of magnetic energy (heating) to be  $10^4$ s. Crust is healed after some time  $\sim 1$ yr
- **Thermal response:** include magnetic heating, Ohmic heating, heat diffusion and neutrino cooling
- We model non-thermal emission from the untwisting of the magnetic field lines in the magnetosphere and measure both thermal and non-thermal fluxes





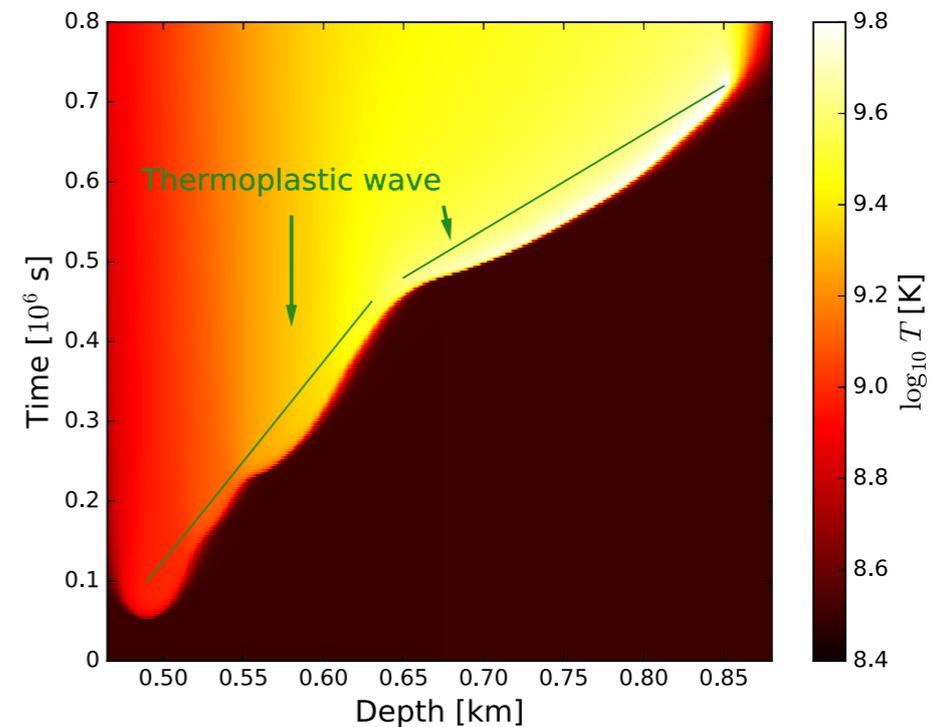
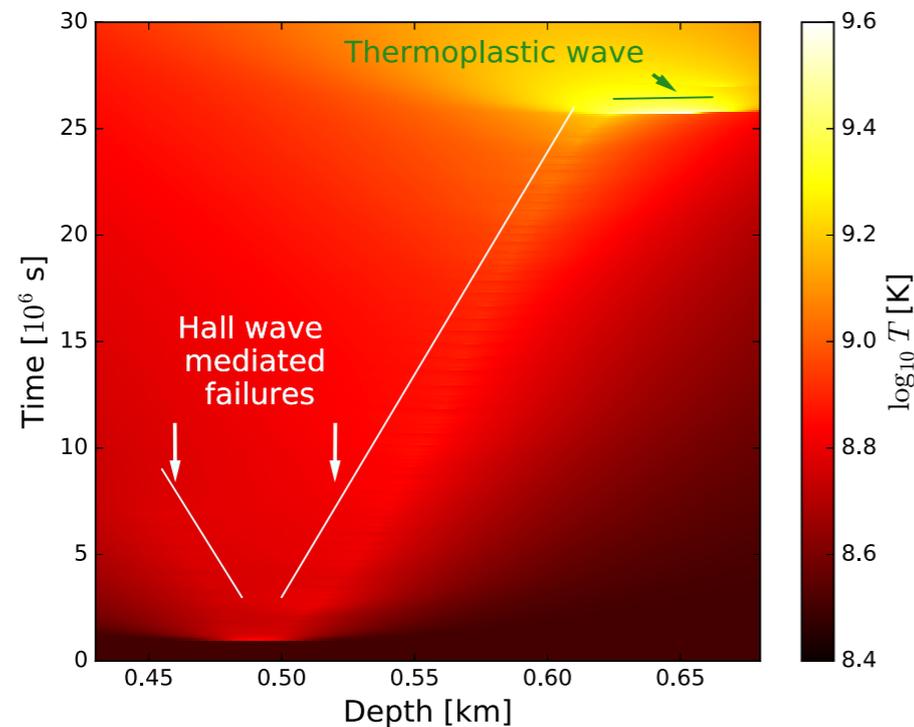
# Hall-mediated Avalanches vs Thermoplastic Waves

- Hall-mediated mode has speed  $v \sim \sqrt{\alpha \frac{B_z c}{4\pi n_e e}}$
- Much smaller than the speed of TPW  $v \sim \sqrt{\alpha \chi}$

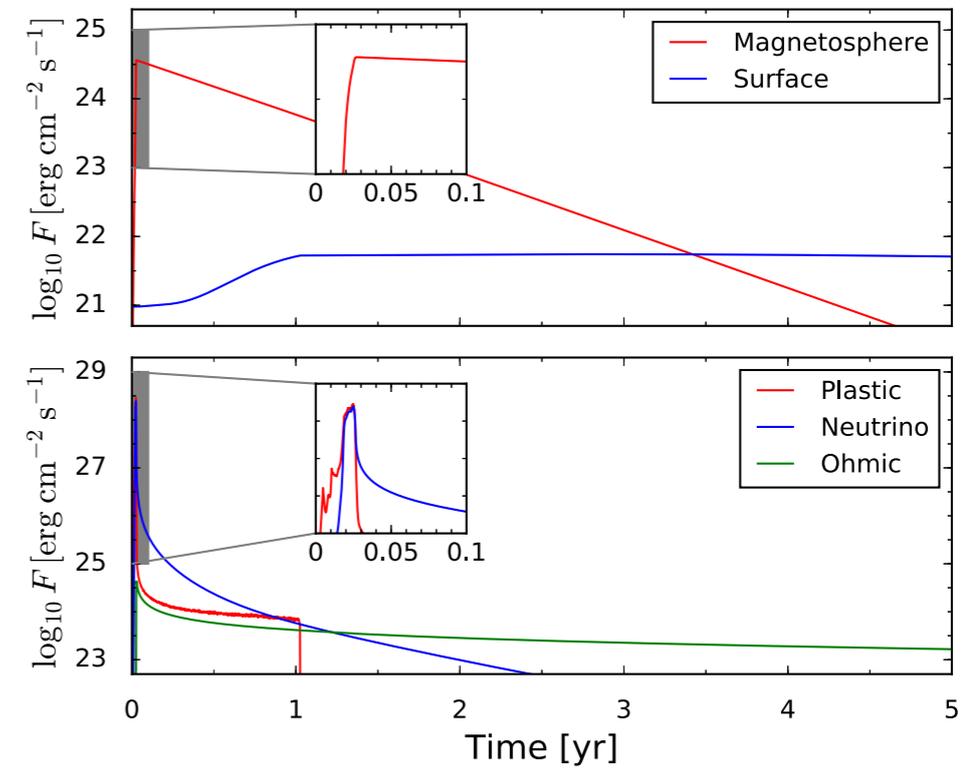
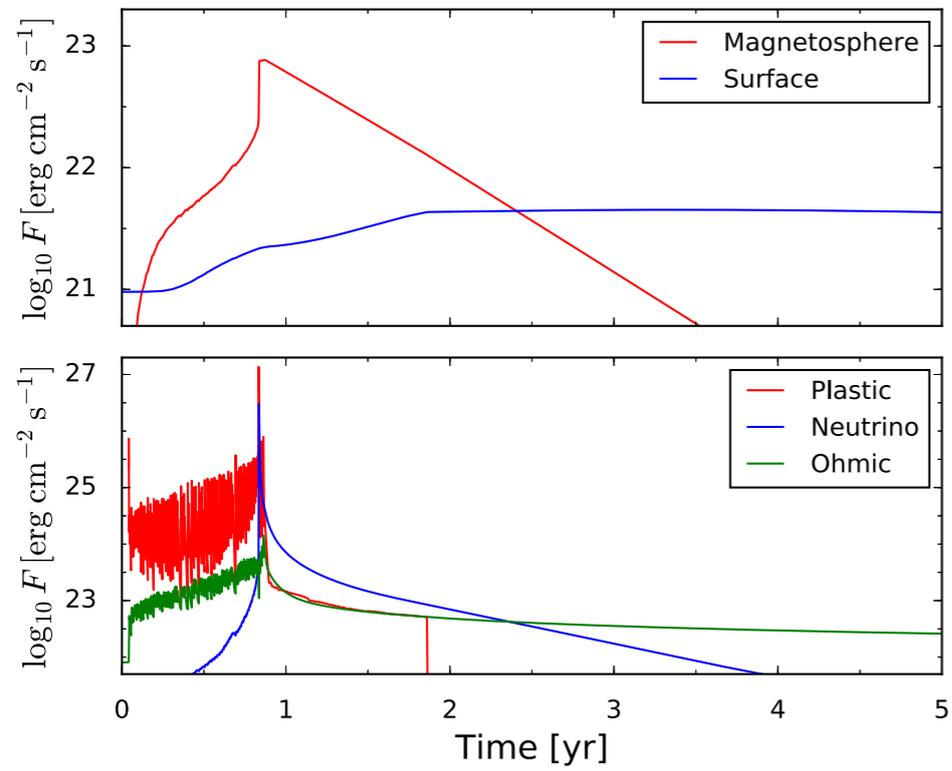


# Hall-mediated Avalanches vs Thermoplastic Waves

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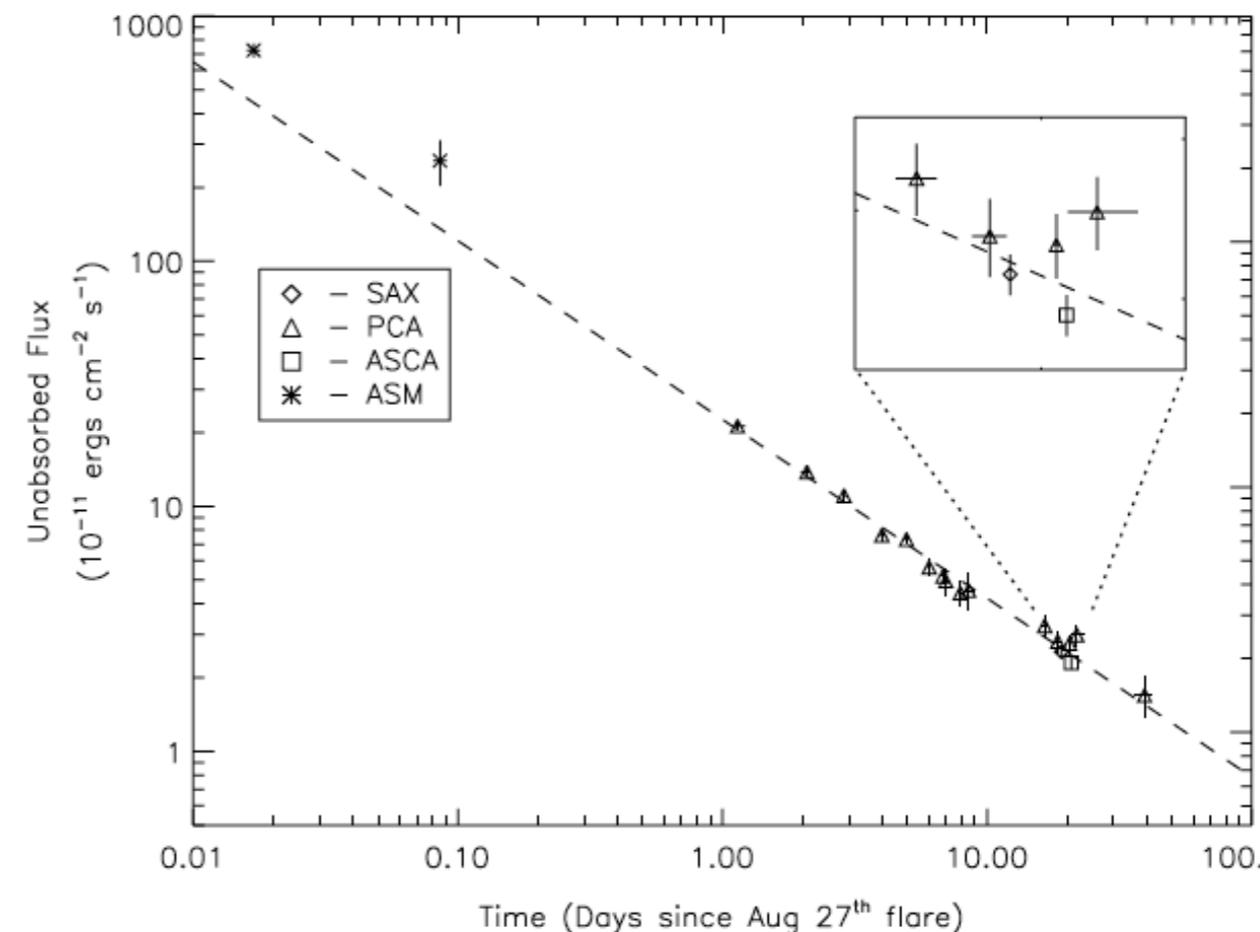


# Light Curves



# Plastic Damping of Alfvén Waves in the Magnetar

- Plastic failures can also be driven by external waves. e.g. Alfvén waves are generated in the giant flares (Parfrey 2013).
- The wave energy is dissipated and heat up the crust.
- Such crustal heating can give rise to thermal afterglow after giant flares. e.g. SGR 1900



Woods et al. (2001)

# Plastic Damping of Alfvén Waves in the Magnetar

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- Waves trapped along closed field lines bounce between magnetic footpoints.
- Part of the wave will be transmitted into the magnetar when they hit the stellar surface.
- We run 1D simulation following the dynamics of Alfvén waves from the magnetosphere into the crust.



NASA/CXC/M.Weiss

# Alfven Wave Dynamics Inside the Magnetar

- Alfven speed inside the magnetar

$$v^2(z) = \frac{B_z^2/4\pi + \mu(z)}{B_z^2/4\pi c^2 + \rho(z)}.$$

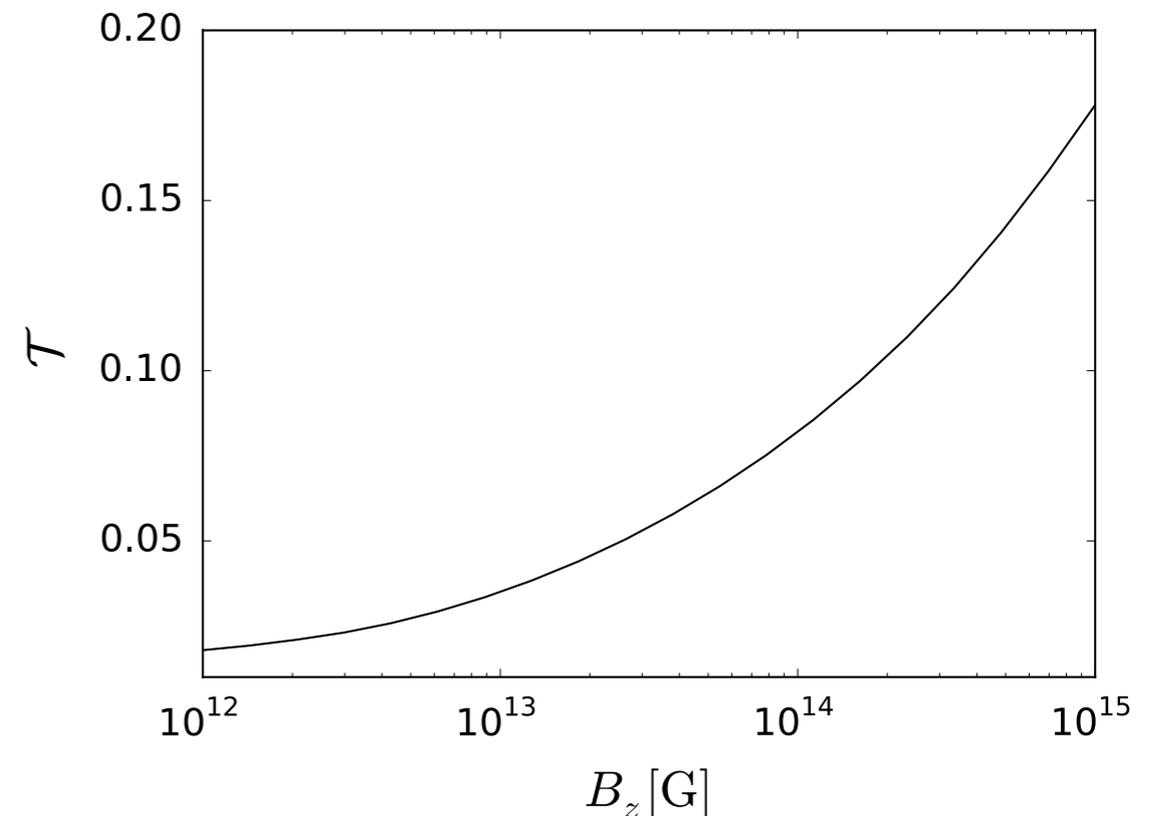
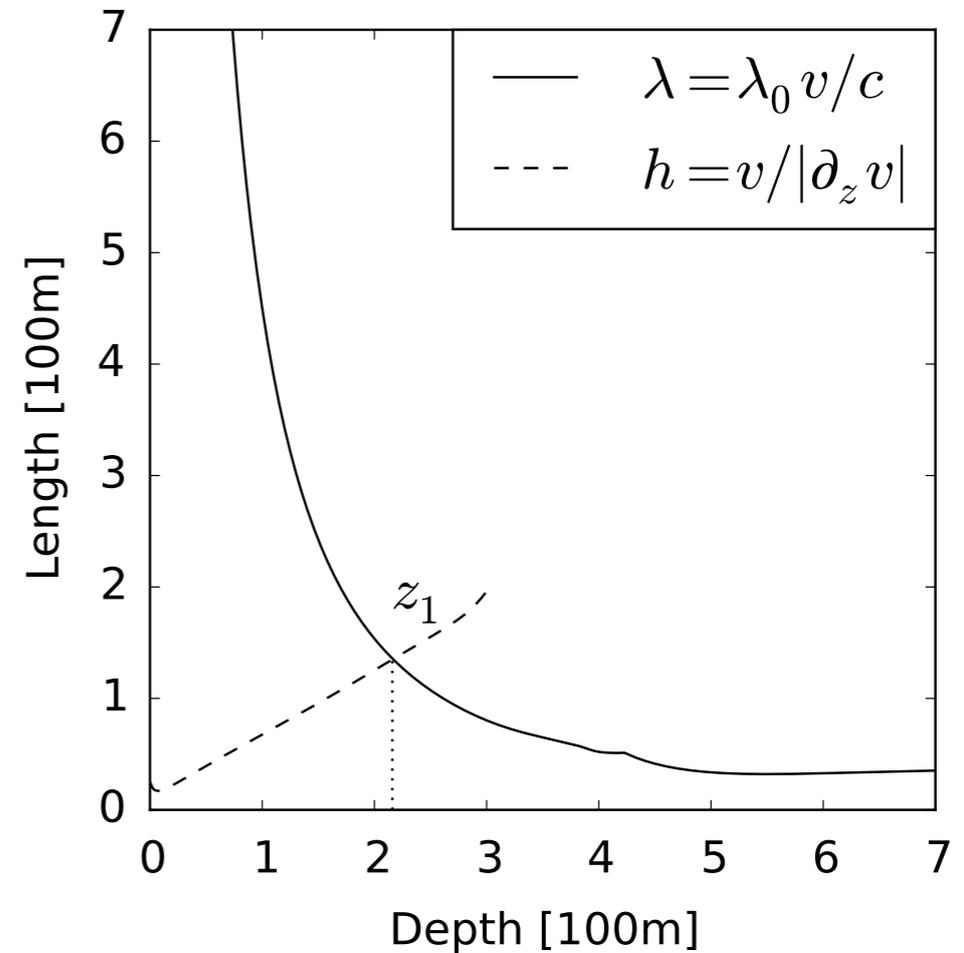
- The transmission coefficient is found to be 10%-20%

$$\mathcal{T} \sim \frac{4Z(z_1)Z(0)}{[Z(z_1) + Z(0)]^2} \approx \frac{4v(z_1)}{c}.$$

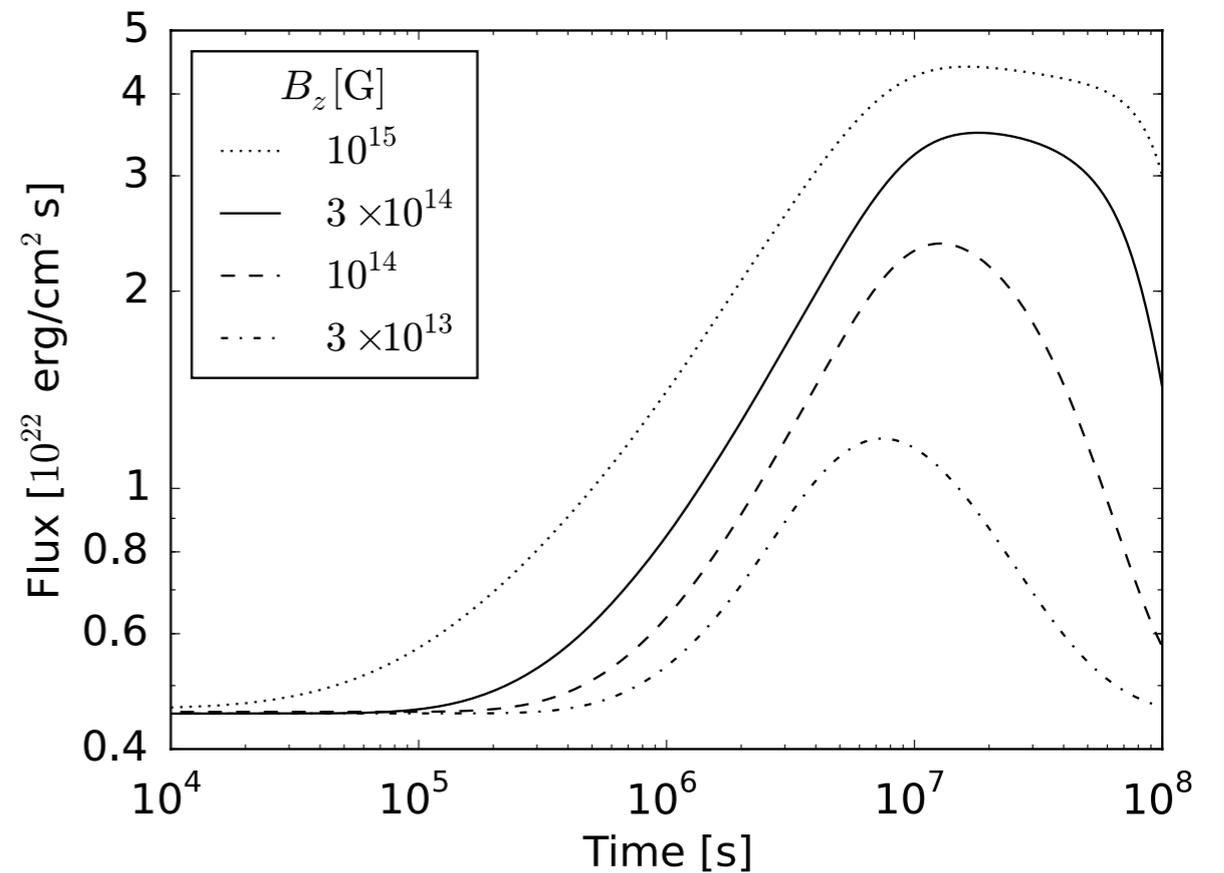
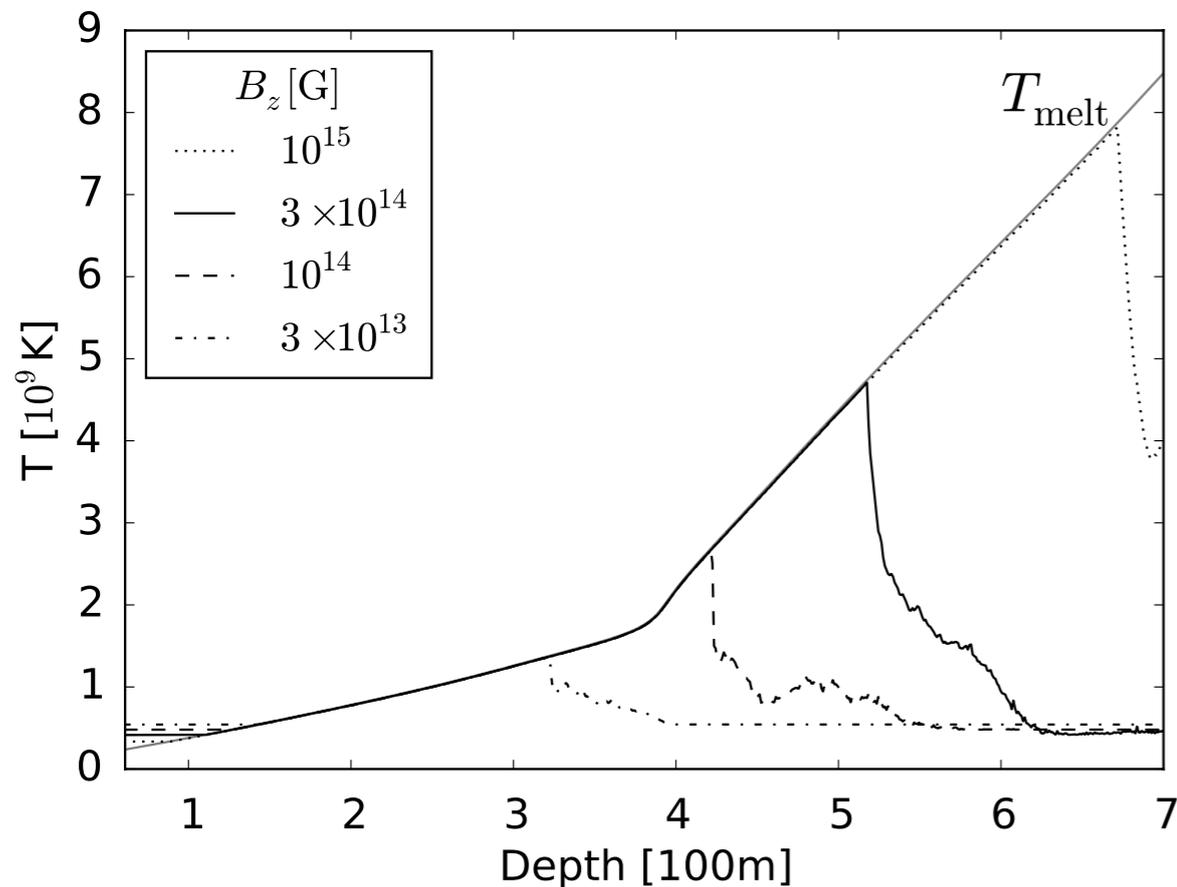
- WKB applies in the inner crust

$$s \propto \rho^{1/4}$$

- The strain increases with density!



# Temperature Profile and Light Curve



- No plastic flow if the crust is melted  $T_{\text{melt}} \approx 2.4 \times 10^9 \rho_{12}^{1/3} \text{ K.}$
- Most heat is lost to neutrino emissions.
- The peak luminosity will saturate. Increasing heating will not increase the thermal luminosity!

- The crustal absorption of Alfvén waves is competed by the magnetospheric dissipation outside the magnetar.
- Thompson & Duncan (1995, 2001): dissipation of Alfvén waves forms an optically thick plasma “fireball” that powers the emission.
- The luminosity of plasma “fireball” emission depends on how much energy ends up dissipated in the magnetosphere.

# Magnetospheric Dissipation

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- **How does magnetospheric dissipation take place?**
- Alfvén waves can lose energy through
  1. Turbulent dissipation from nonlinear interactions
  2. Conversion to fast waves that can escape the magnetosphere
  3. Magnetic reconnection?
  4. QED shocks (Heyl and Hernquist 1998)?

# Force-Free Electrodynamics (FFE)

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- Magnetic energy dominates over the rest mass energy of the plasma
- The plasma follows the field dynamics with a vanishing Lorentz force  $\rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$ ,
- The equation implies two force-free conditions

$$E < B$$

$$\mathbf{E} \cdot \mathbf{B} = 0$$

# Waves and Interactions in FFE

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- Alfvén waves  $\omega = |k_z|$
- Fast waves  $\omega = |k|$
- Three-wave interactions are not possible for

$$A + A \rightarrow A$$

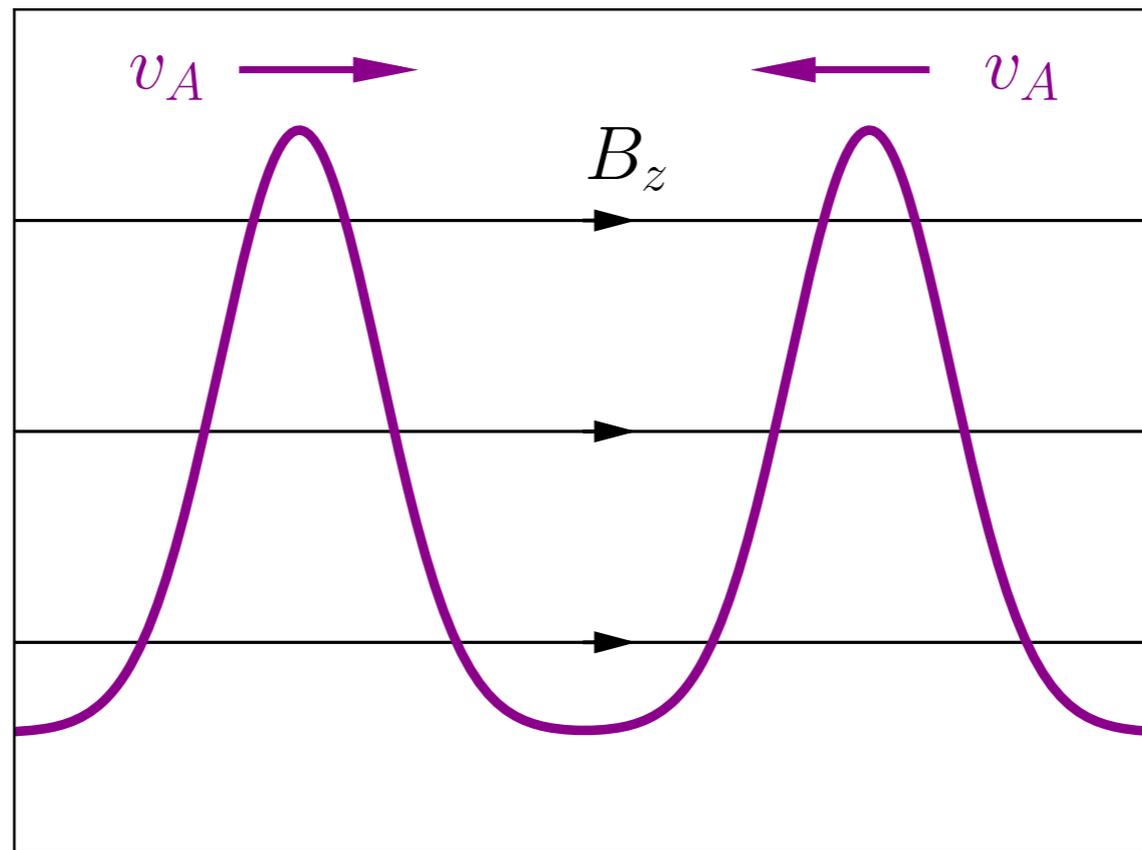
$$F + F \rightarrow F/A$$

- $A + A \rightarrow F$  is a valid channel (Thompson & Blaes 1998)

# Simulation Set-up

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- We simulate collision of a pair of counter-propagating Alfvén wave pulses in a periodic Cartesian box.



# Numerical Methods

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- Very high order scheme: 5th order WENO + Roe solver
- Hyperbolic divergence cleaning
- $\mathbf{E} \cdot \mathbf{B}$  is dynamically damped (Parfrey 2017)
- $\mathbf{E} \rightarrow \sqrt{\frac{B^2}{E^2}} \mathbf{E}$  . to restore magnetic dominance

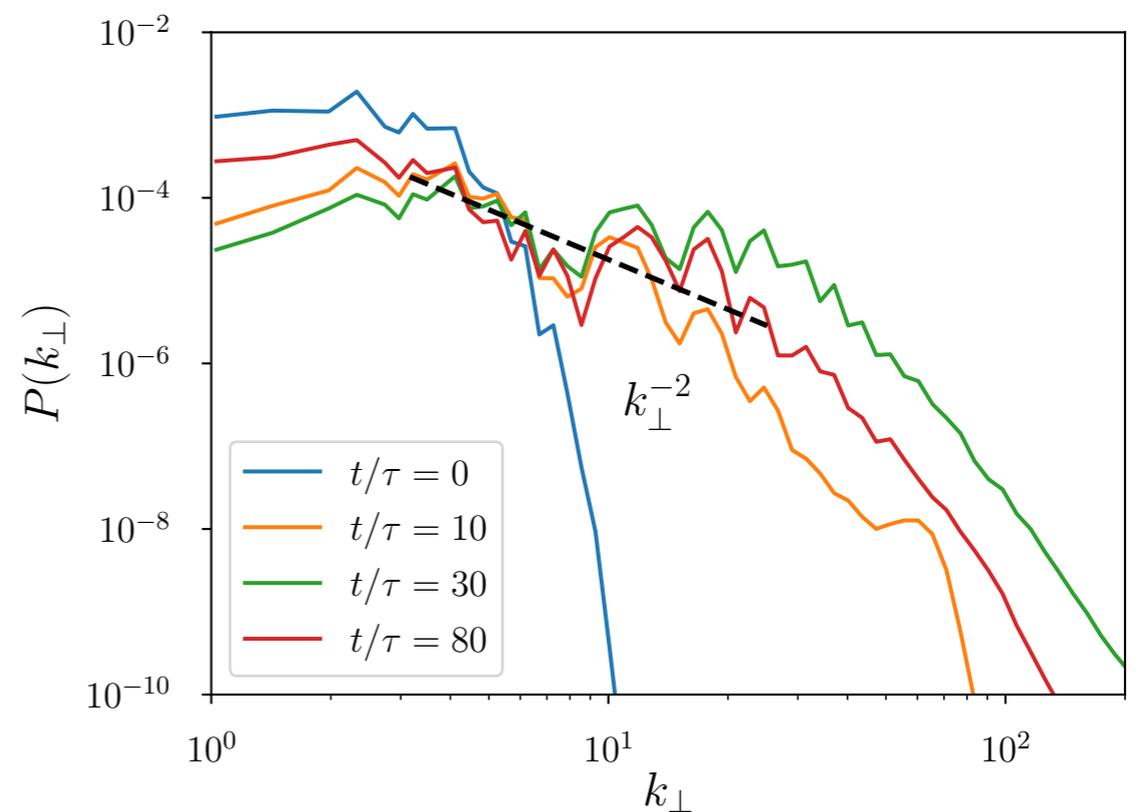
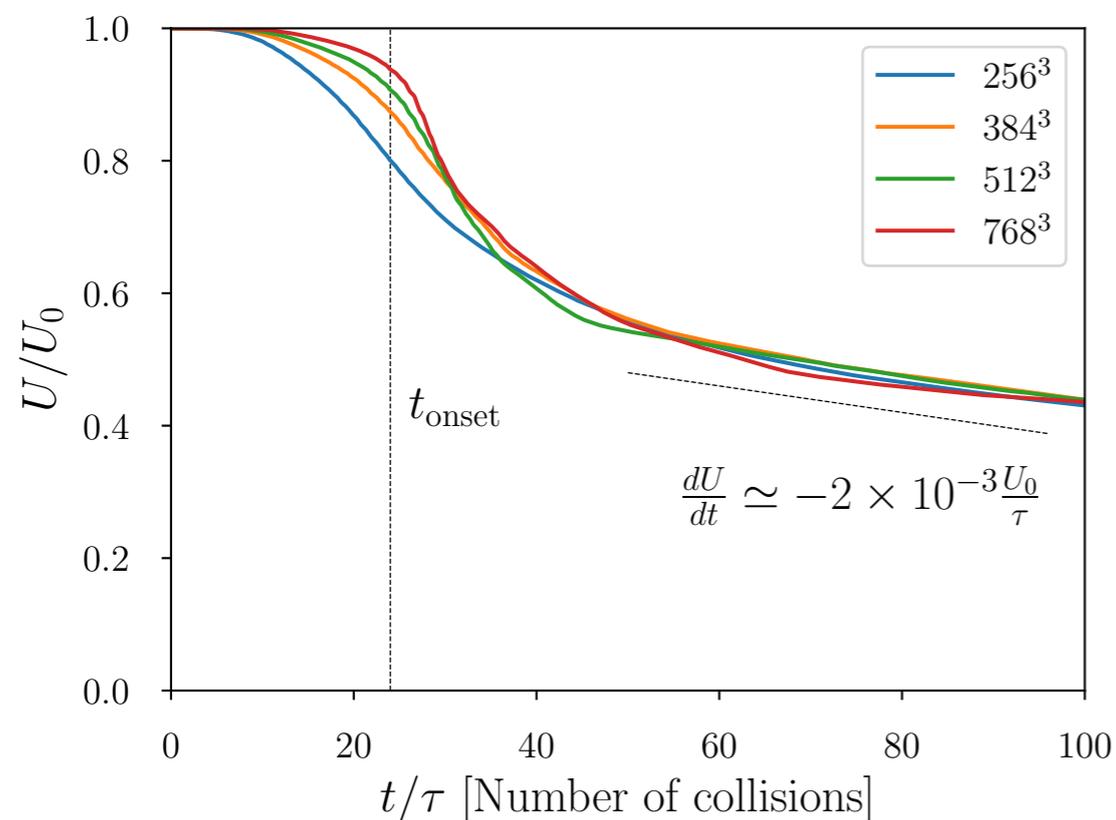
# Numerical Dissipation in the Code

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- 1) Hyperbolic divergence cleaning
- 2) Cleaning of  $\mathbf{E} \cdot \mathbf{B}$
- 3) Reduction of electrical field  $\mathbf{E} \rightarrow \sqrt{\frac{B^2}{E^2}} \mathbf{E}$
- 4) Grid-heating
- Channel 1 and 2 should converge away with increasing resolution
- Channel 4 is expected to capture the true dissipation rate for a turbulent cascade with constant flux
- Channel 3 will be discussed later

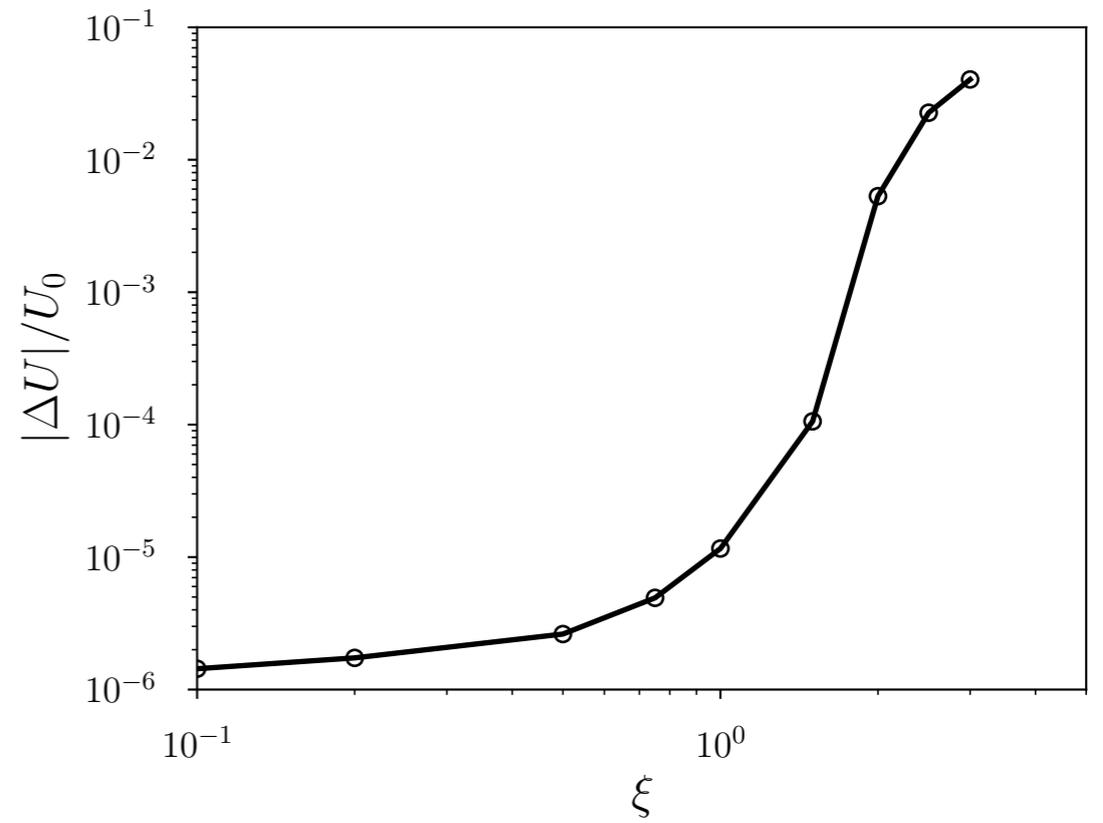
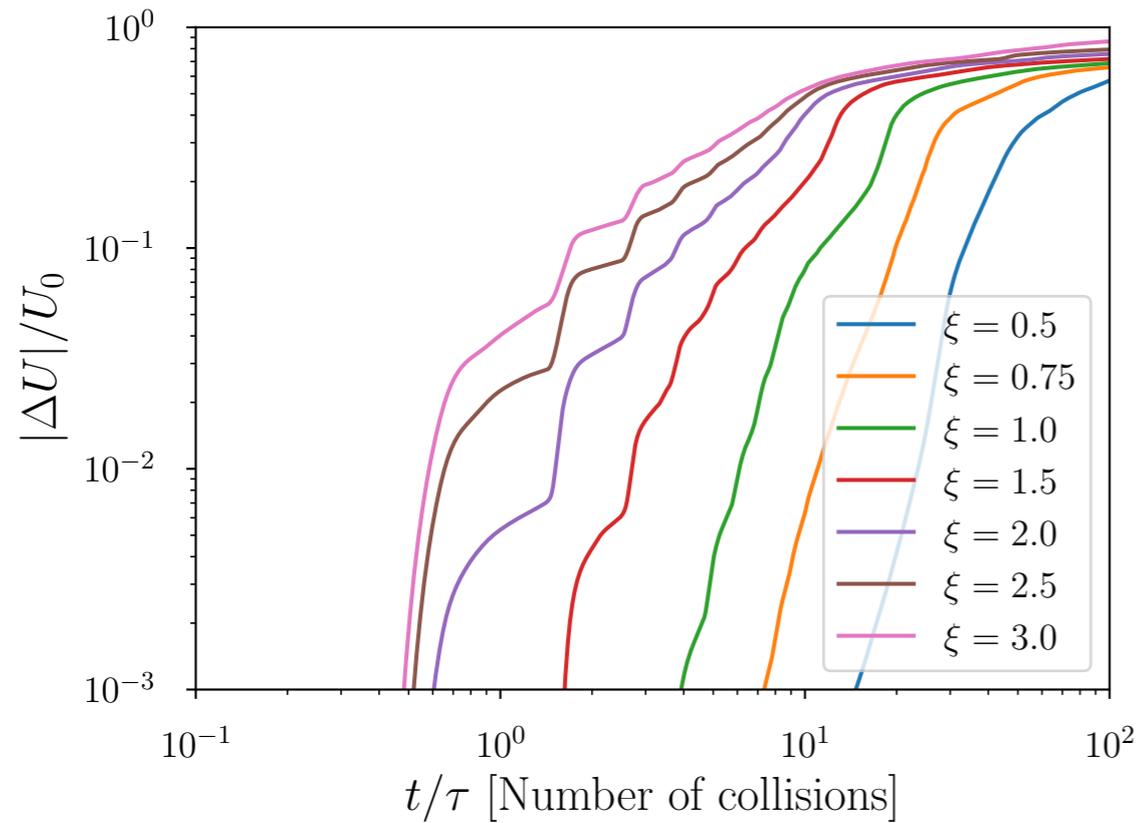
# Development of turbulent spectrum in 3D

- Forward cascade with anisotropic power-law spectrum  $k_{\perp}^{-2}$  is observed
- The dissipation comes from grid heating with a certain onset time independent of grid resolution



# Turbulent dissipation rate

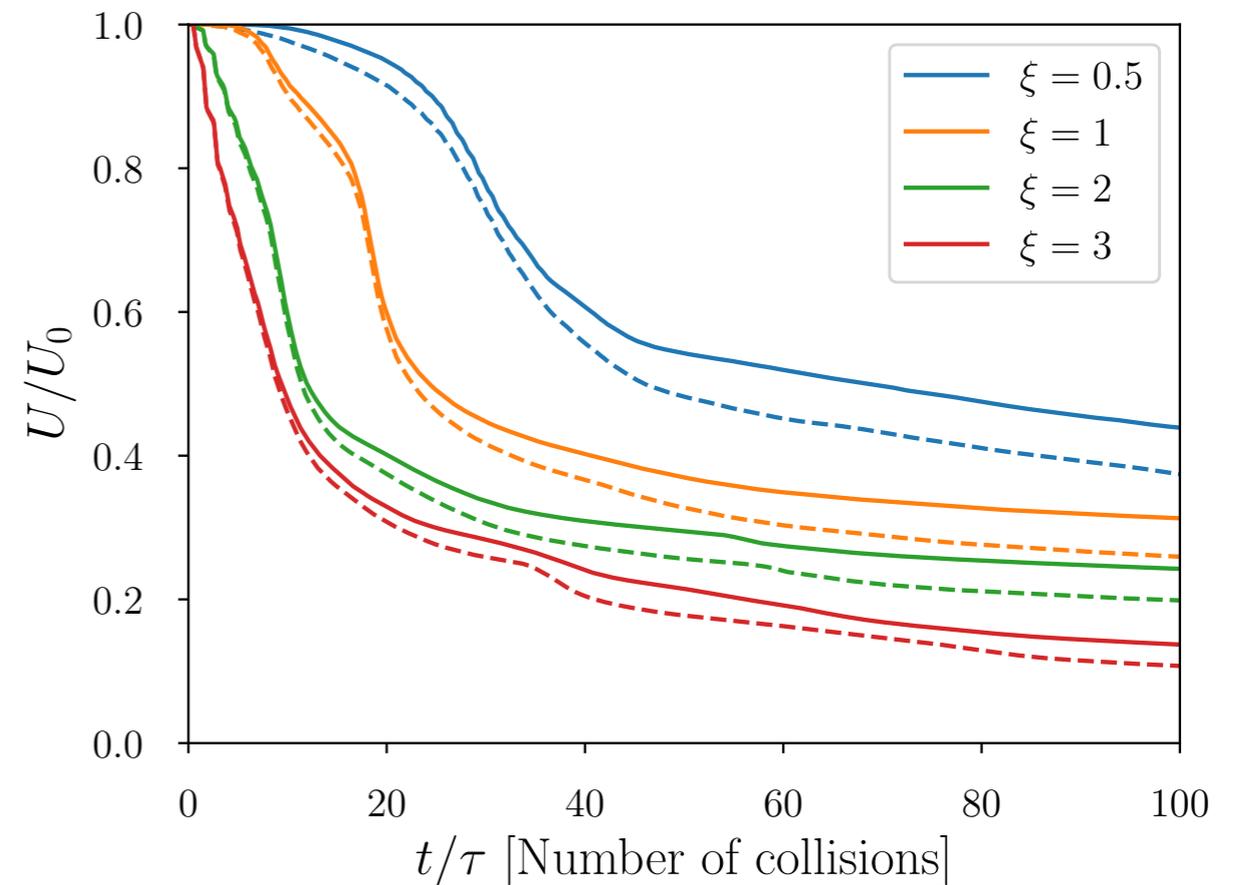
$\xi$ : wave amplitude



# Energy Carried Away by Fast Waves

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- Add ohmic dissipation layer on the boundary of transverse plane to damp wave energy which mimics the energy lost due to the escape of fast waves
- Alfvén waves are confined on the field lines and can not reach the dissipative boundary
- Only fast waves are damped



- Compared with  $\sim 10\%$  transmission rate of Alfvén waves into the magnetar, the magnetospheric dissipation is weak unless the amplitude is much larger than unity.
- Most wave energy is lost inside magnetar and power thermal emission.
- The residue wave energy dissipated in the magnetosphere powers the fireball emission.
- The wave energy can be much larger than the observed radiation since most heat is lost to neutrinos (Beloborodov & Li 2015).

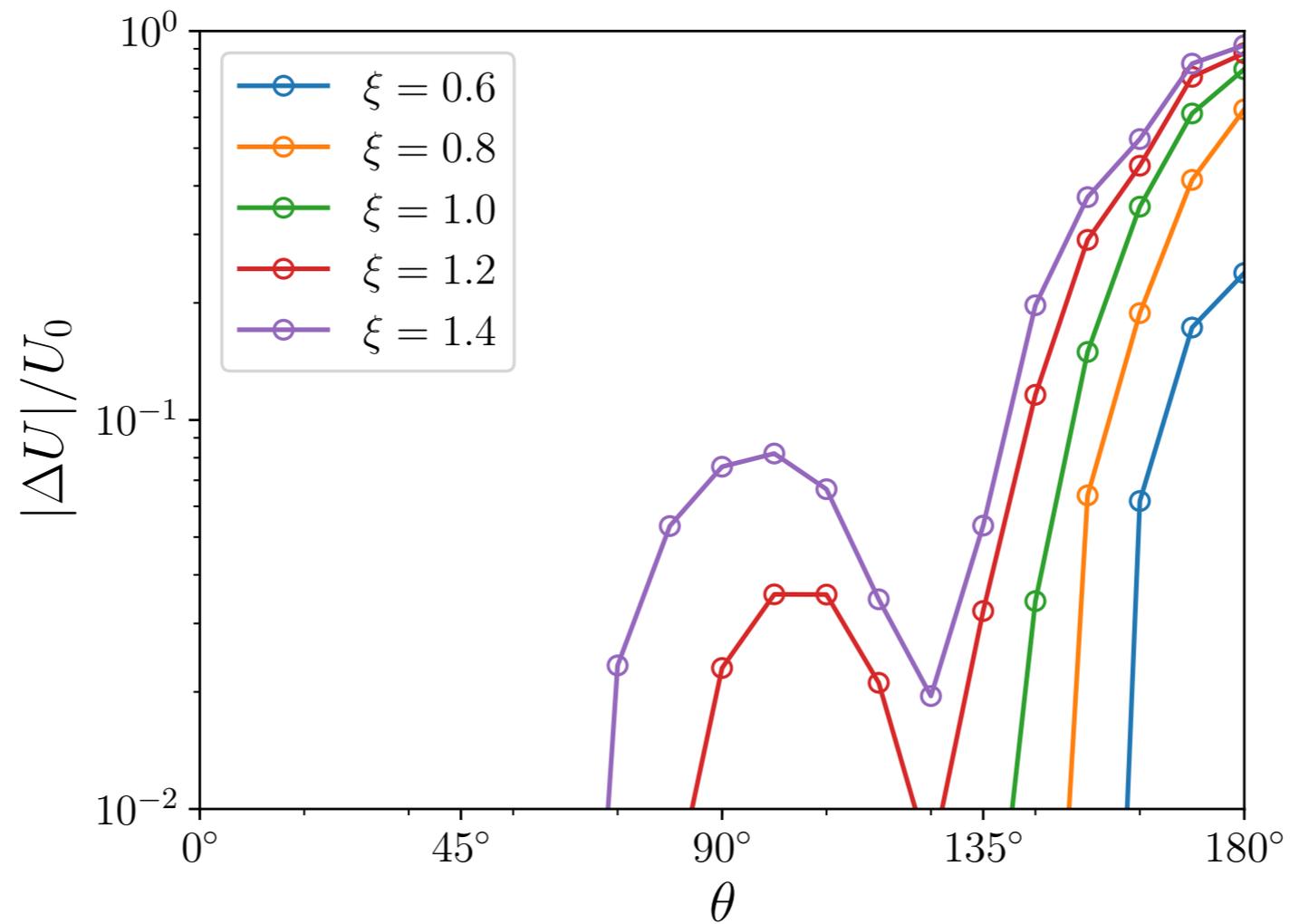
# Immediate Dissipation in FFE

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- All previous simulations are done for Alfvén waves have same polarizations (same direction of B field along a given field line).
- When they are not aligned, nonlinear interaction can induce  $E > B$  and break the force-free condition.
- Usually, one reduces E field by hand and assumes this process simulates the realistic fast dissipation there (McKinney 2006, Spitkovsky 2006).

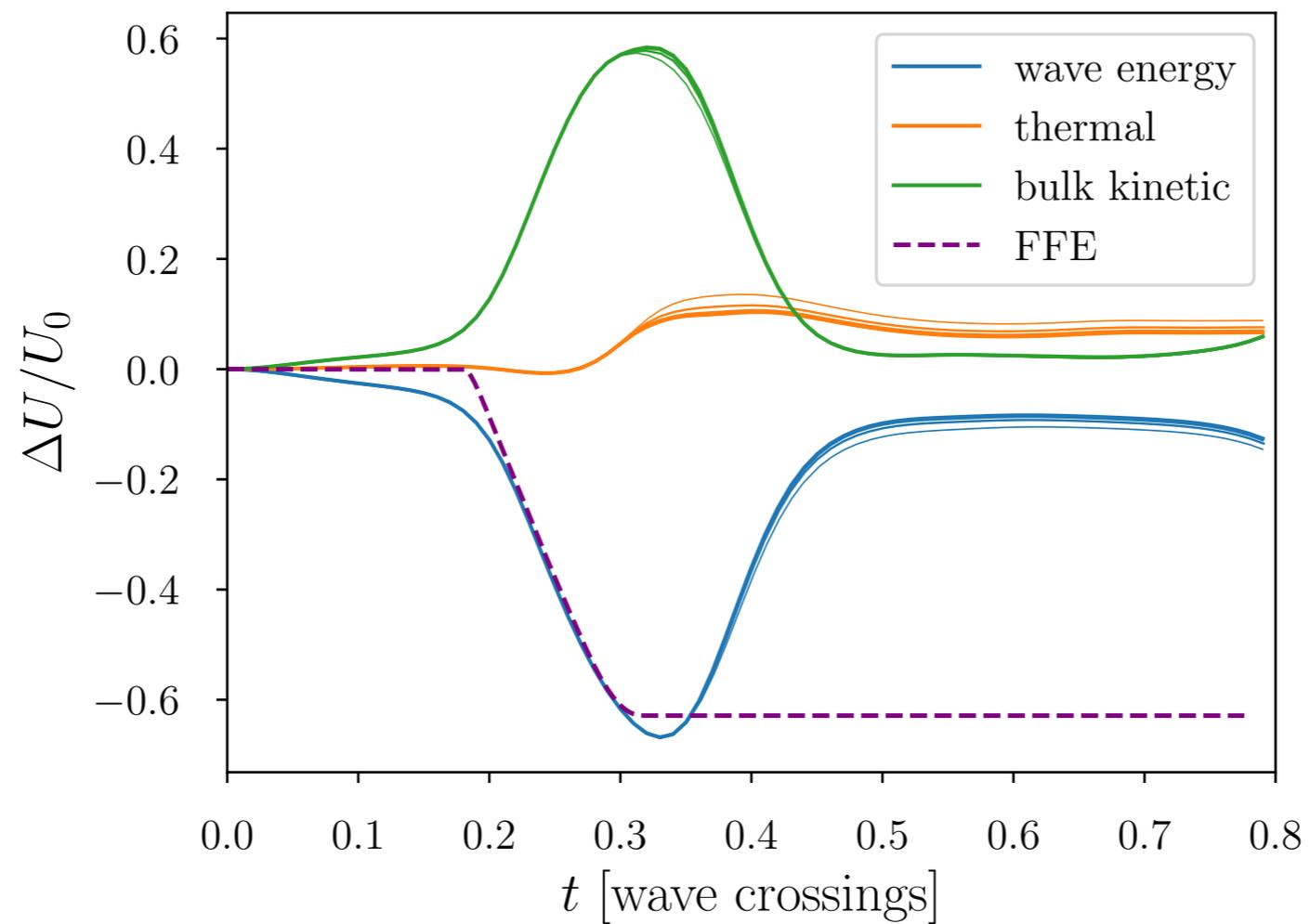
# Energy Loss in a Single Collision for 1D Waves

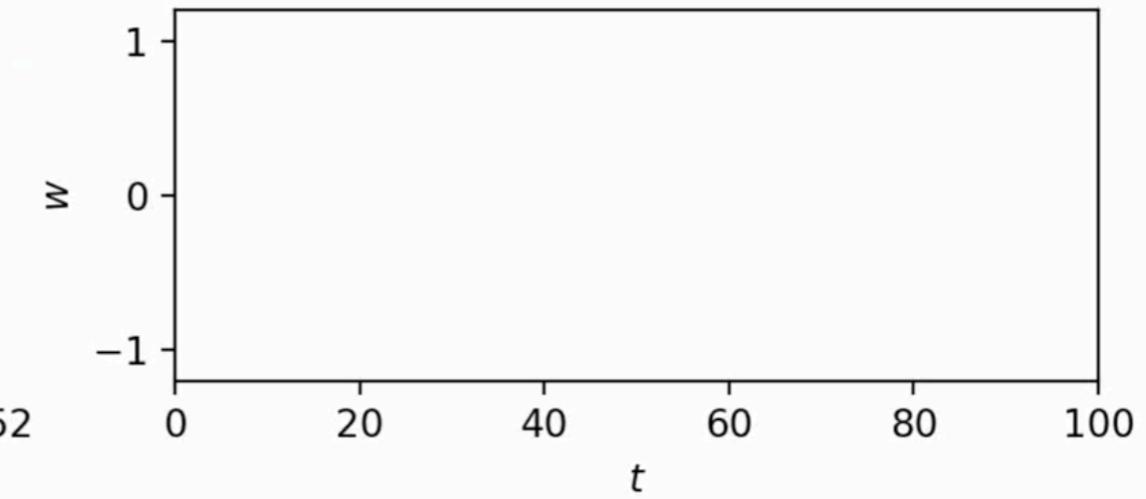
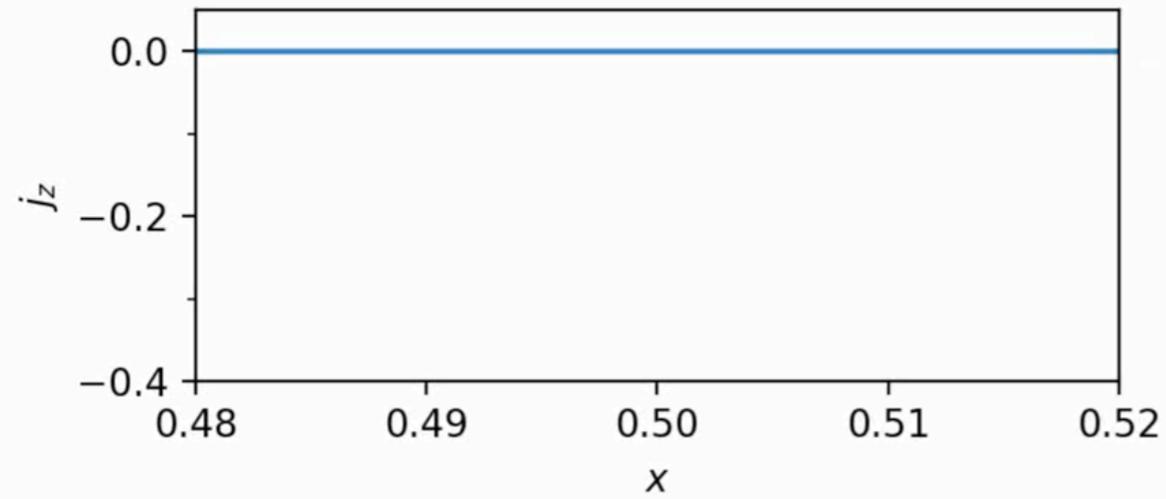
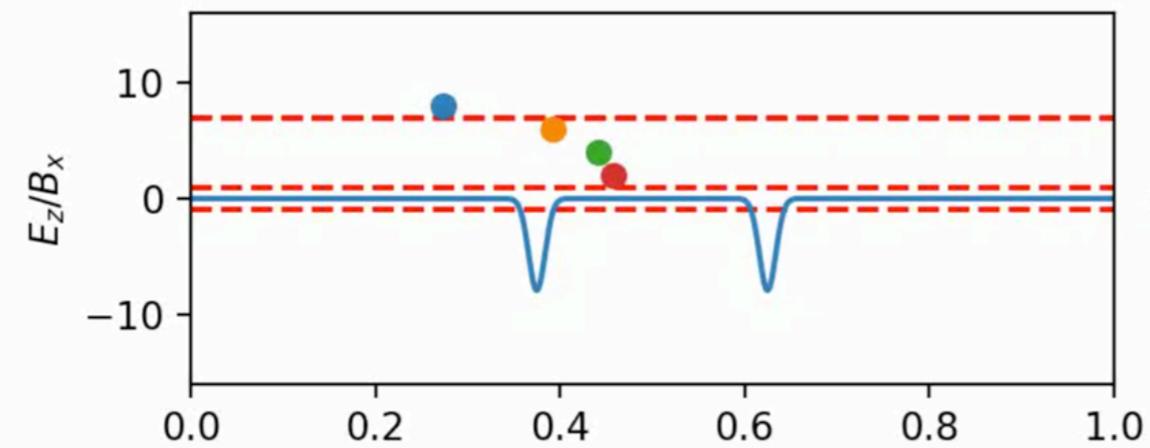
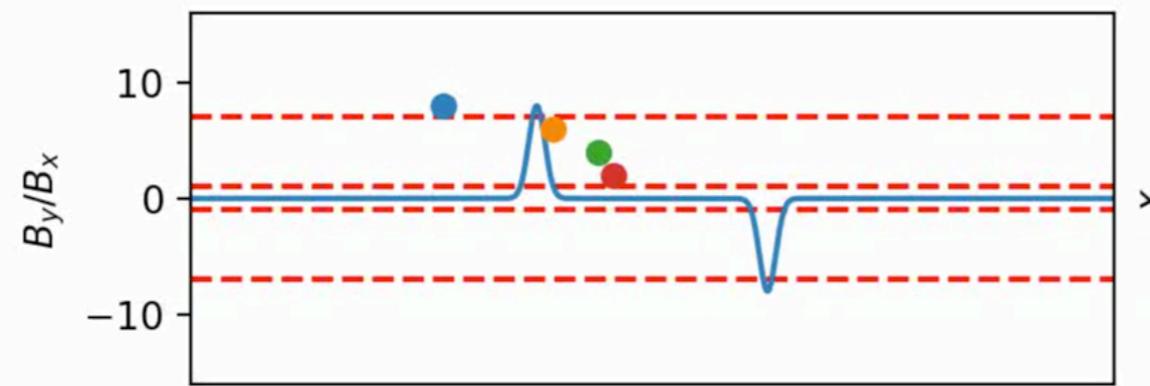
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# Comparison with Relativistic MHD

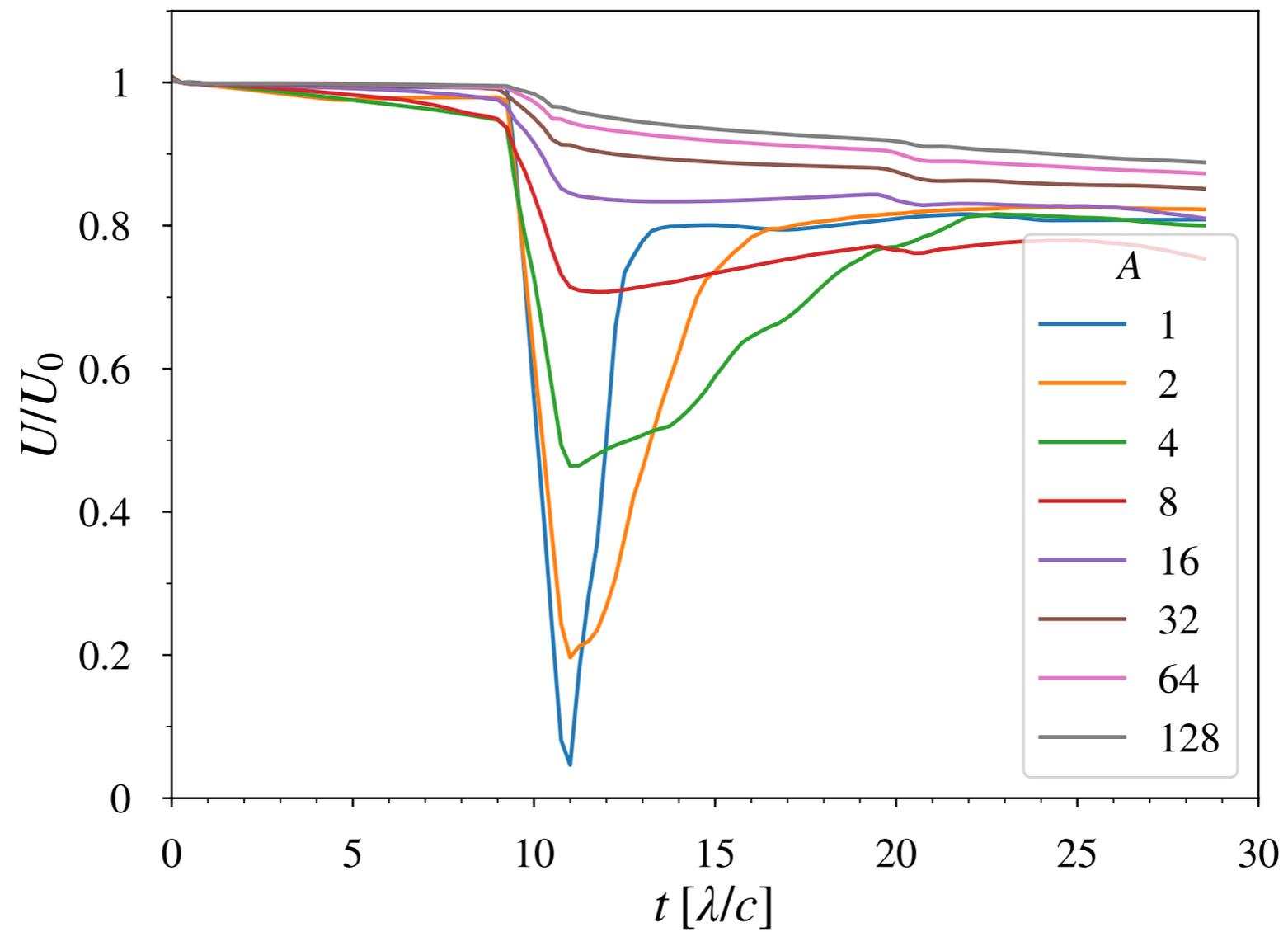
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# PIC Simulations

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# Conclusion

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## **Inside the magnetar:**

- Hall-mediated avalanches and TPW are able to reproduce light curves for outbursts of transient magnetars.
- External Alfvén waves can also trigger plastic flow and lead to thermal afterglow.

## **Outside the magnetar:**

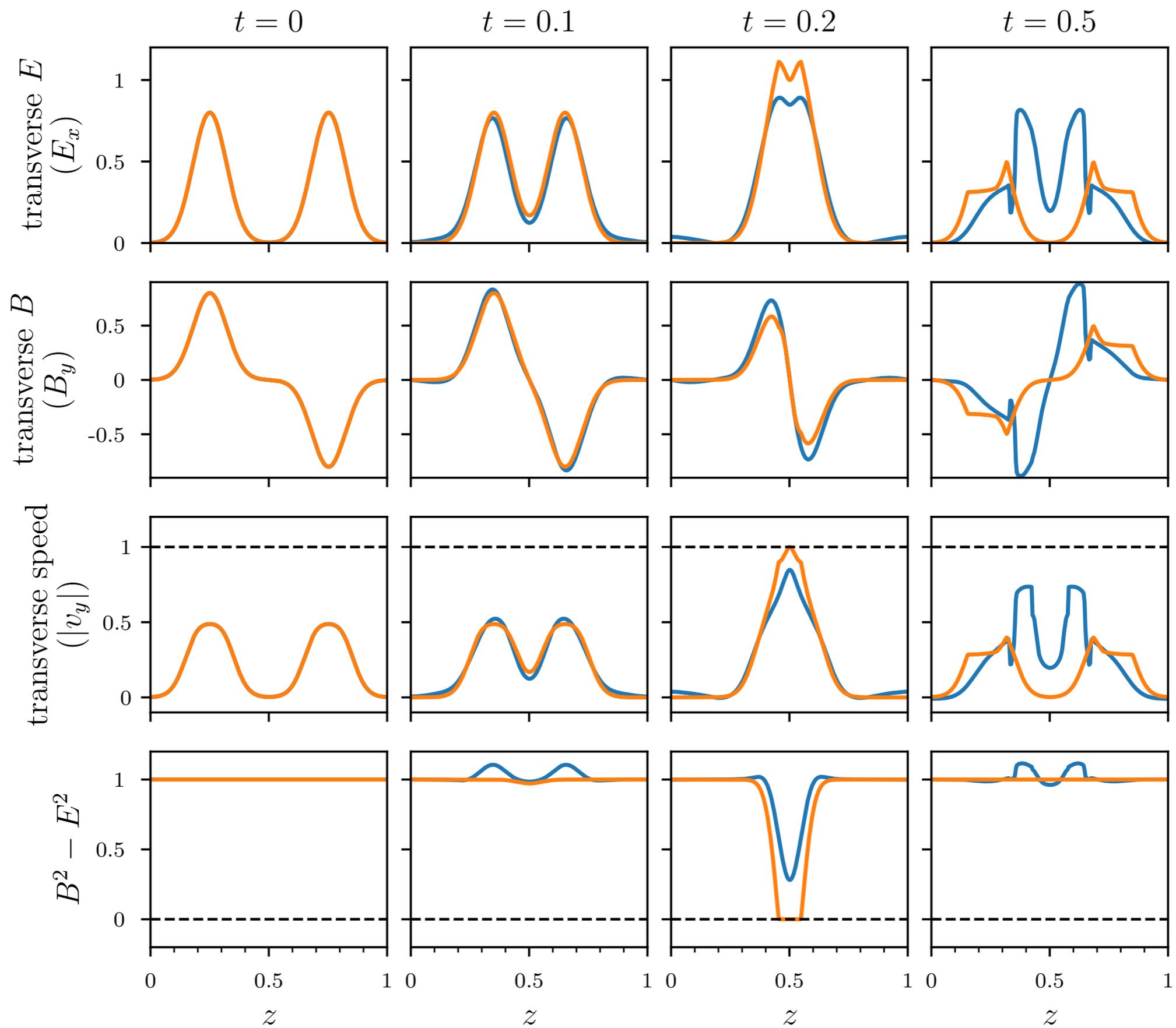
- Loss of wave energy to turbulent dissipation or fast waves is slow
- Most Alfvén wave energy is dissipated inside the magnetar

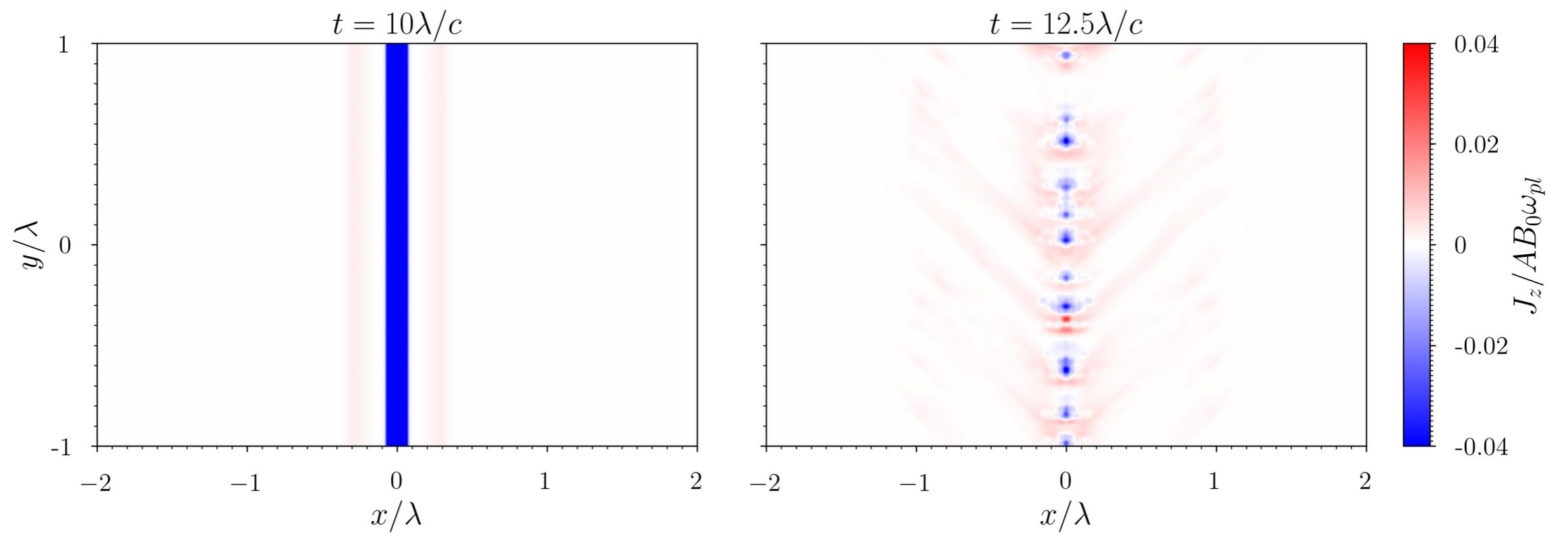
# Future Direction

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- How to correctly model the dissipation when  $E > B$  in FFE?
- Is there magnetic reconnection in those regions?  
(ongoing, PIC simulations)
- 3D simulation of wave interactions with realistic geometry  
(ongoing, cubed sphere mesh)
- Simulation that couples the crust and the magnetosphere
- Connection to FRB, GRB and NS-NS merger (double magnetar)?

Supplementary

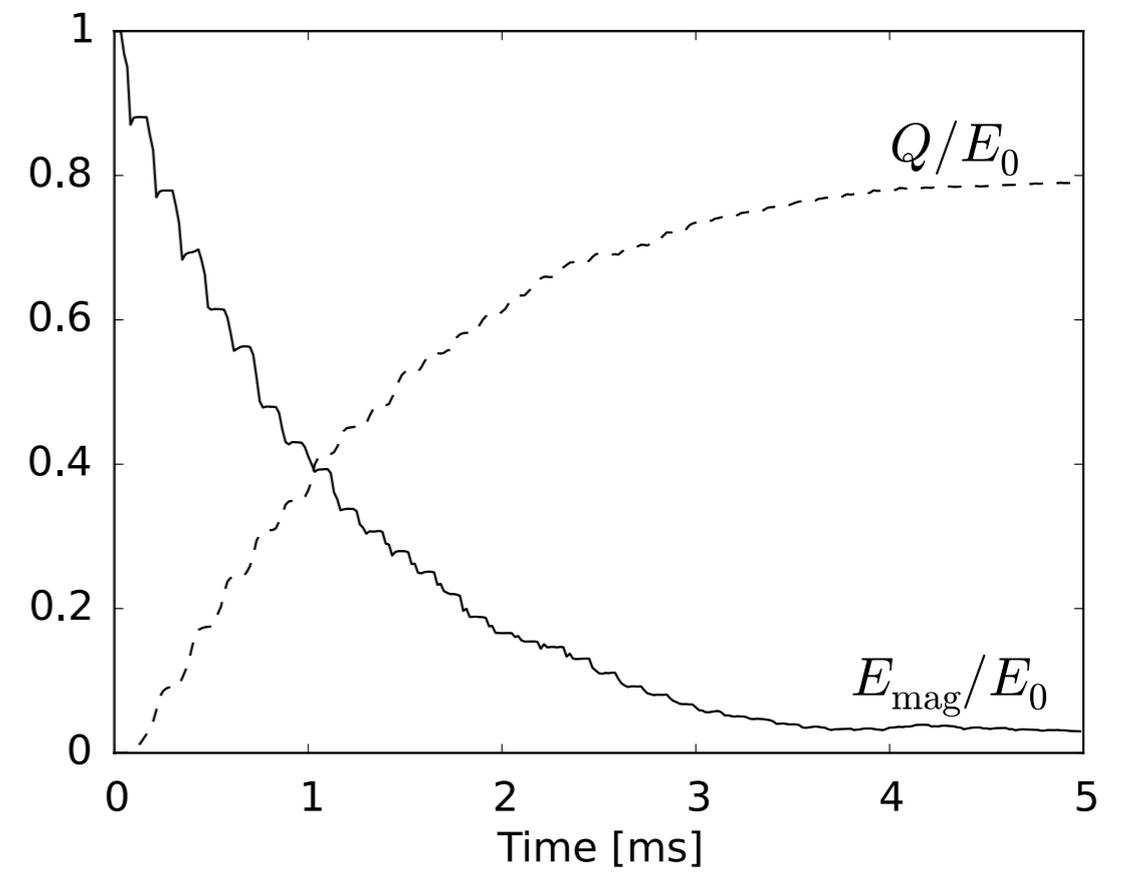
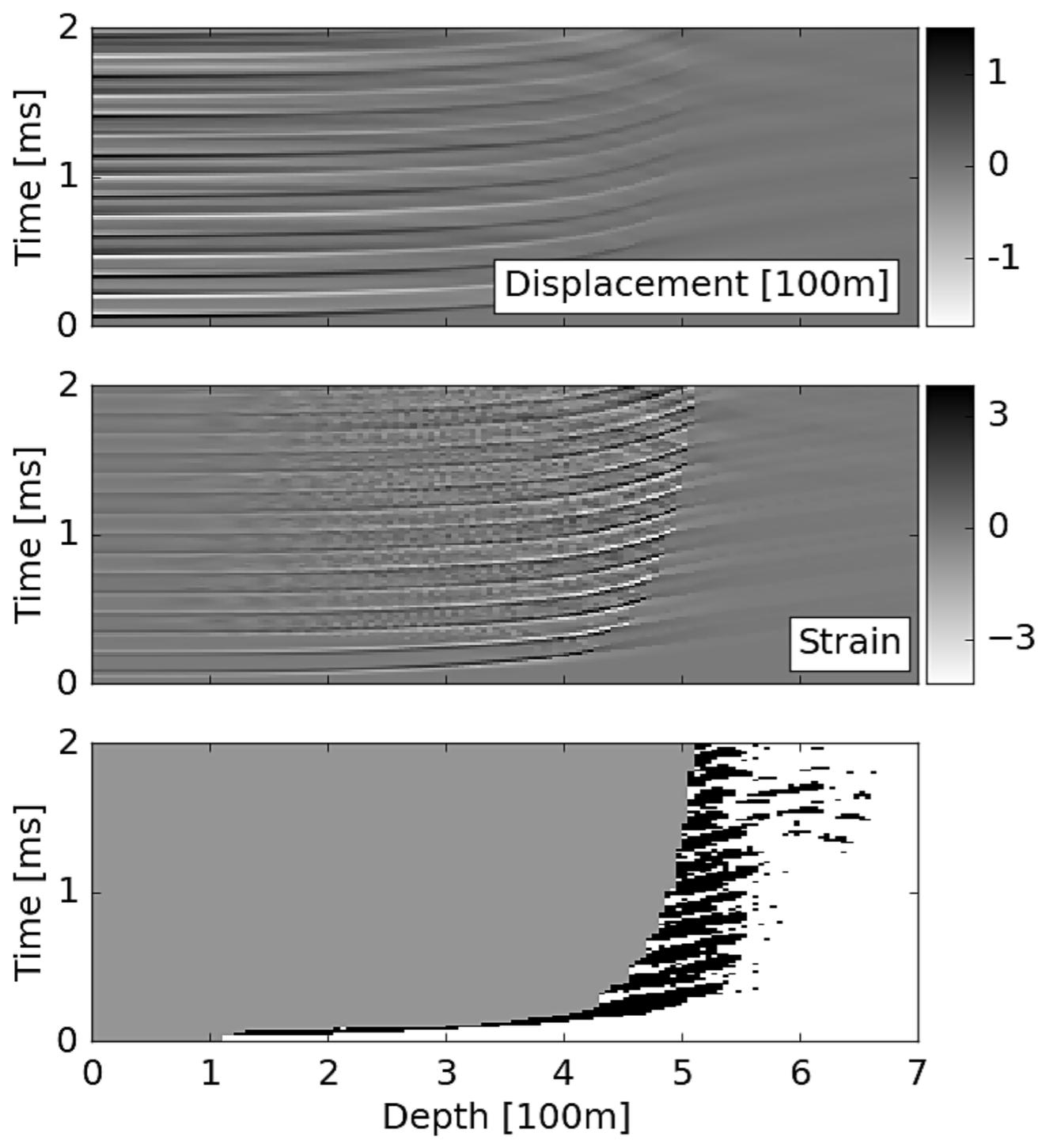




$$\dot{B} = -i\partial_z (D\partial_z B) + B_z\partial_z\dot{\xi}.$$

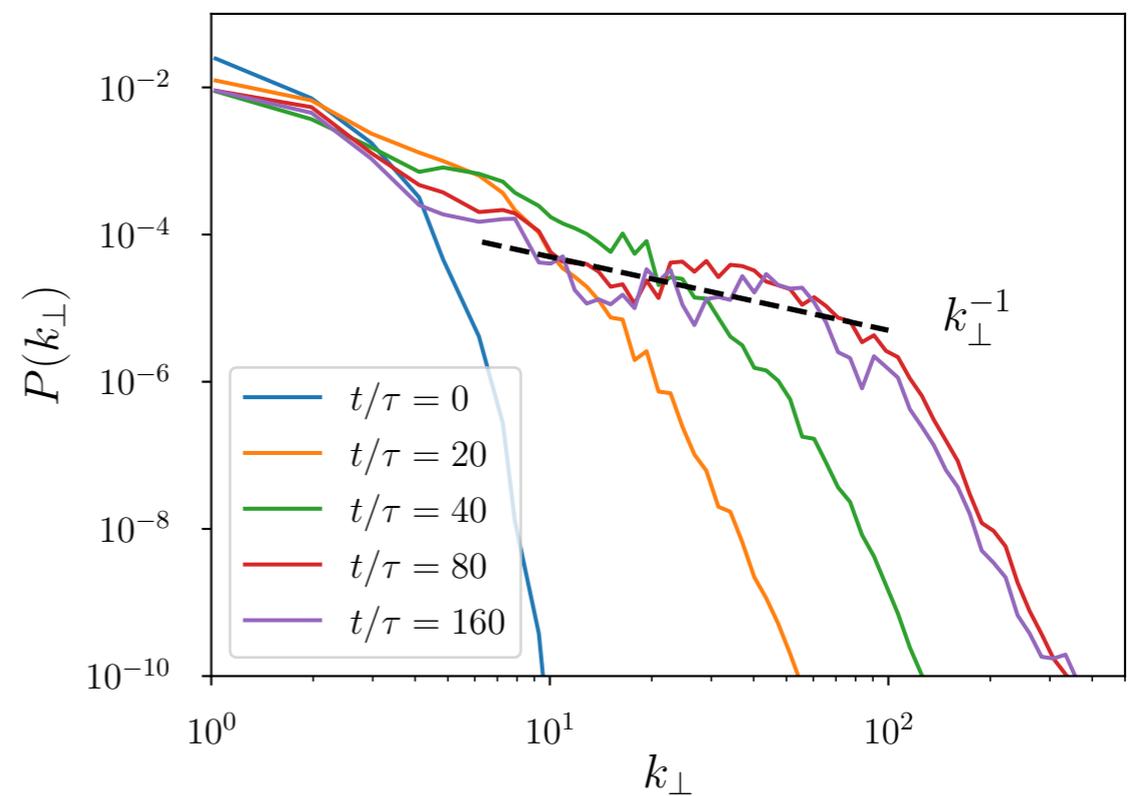
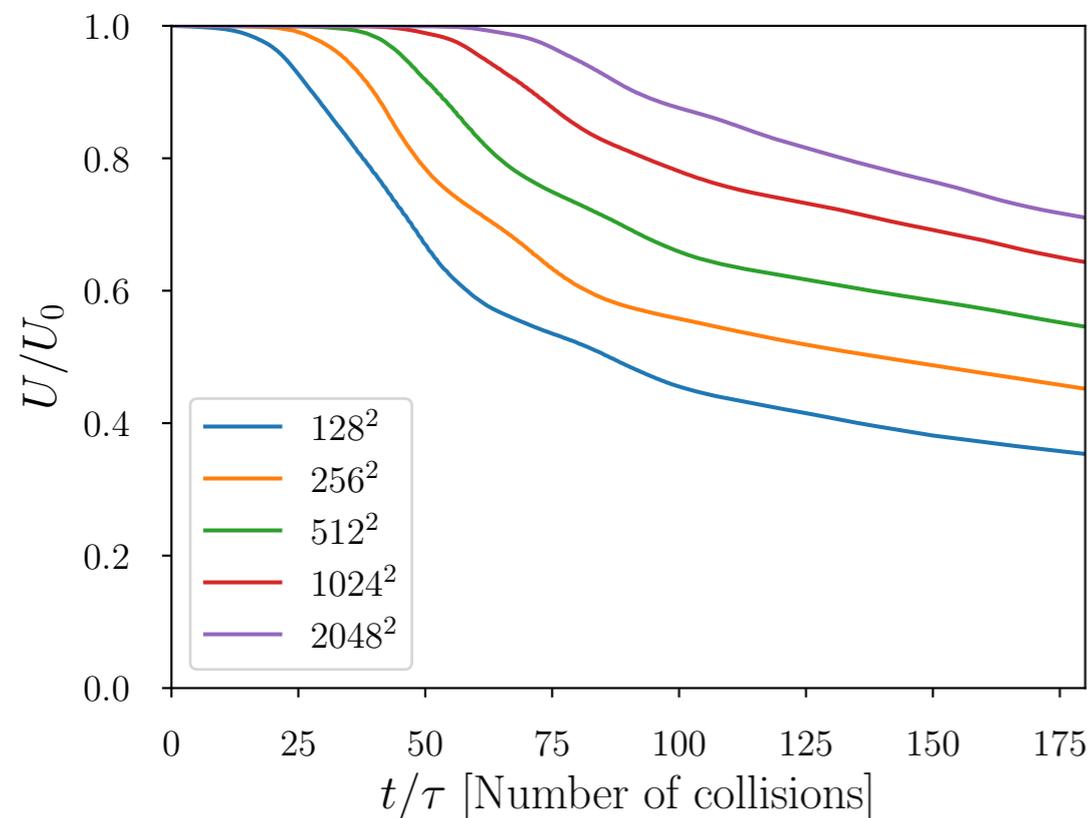
$$B \equiv B_x + iB_y$$

$$D \equiv \frac{B_z c}{4\pi n_e e} + i \frac{c^2}{4\pi\sigma_{\text{ohm}}}$$



# Non-Kolmogorov spectrum in 2D

- When dissipation occurs depends on the resolution
- A shallow spectrum  $k_{\perp}^{-1}$  is observed



# The particularity of shallow spectrum

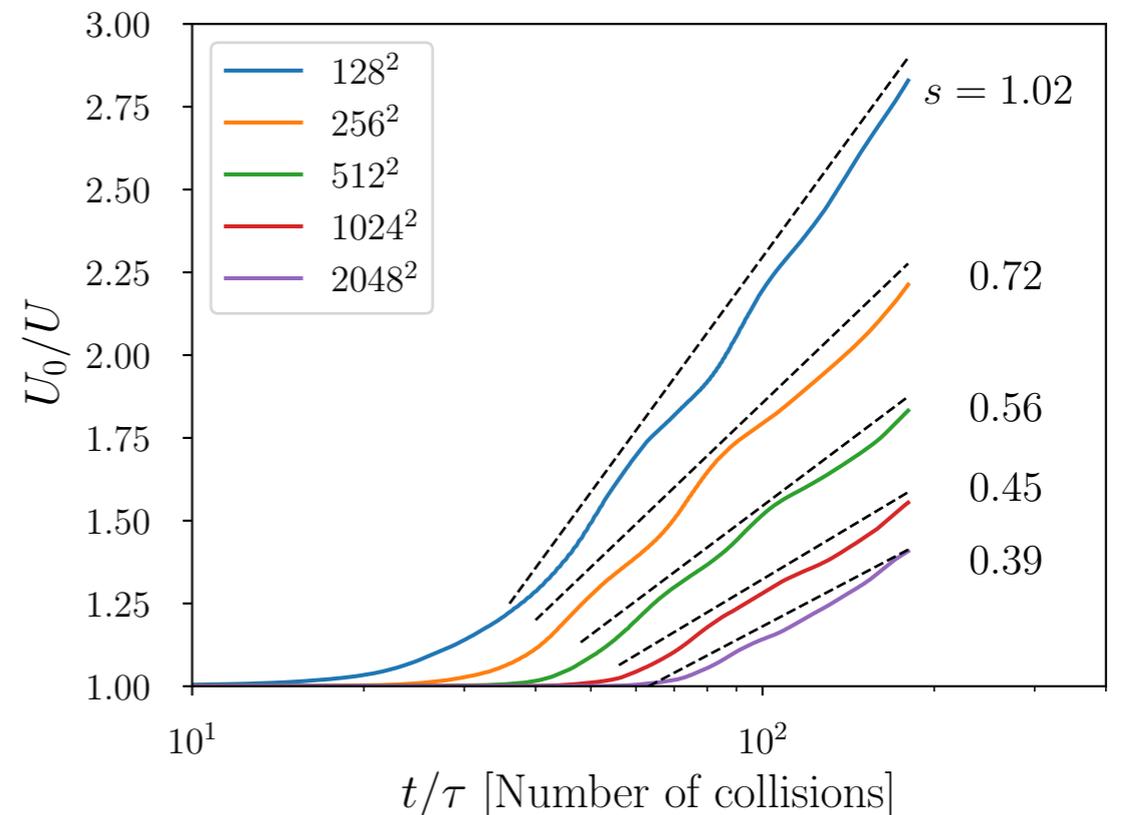
- Energy stored in the spectrum tail is infinite

$$U \propto \int dk_{\perp} k_{\perp}^{-1} \rightarrow \infty$$

- When energy cascades to small scales,  $k_{\max}$  grows polynomially  $t^q$  in time, unlike the 3D case where  $k_{\max}$  diverges at finite time
- We can model this and find

$$\frac{U_0}{U(t)} = \frac{q \log t + \beta}{\log(k_{\text{diss}}/k_0)}$$

with  $q \approx 1.75$



# Transmission coefficient

- Reflection takes place when

$$\lambda > h$$

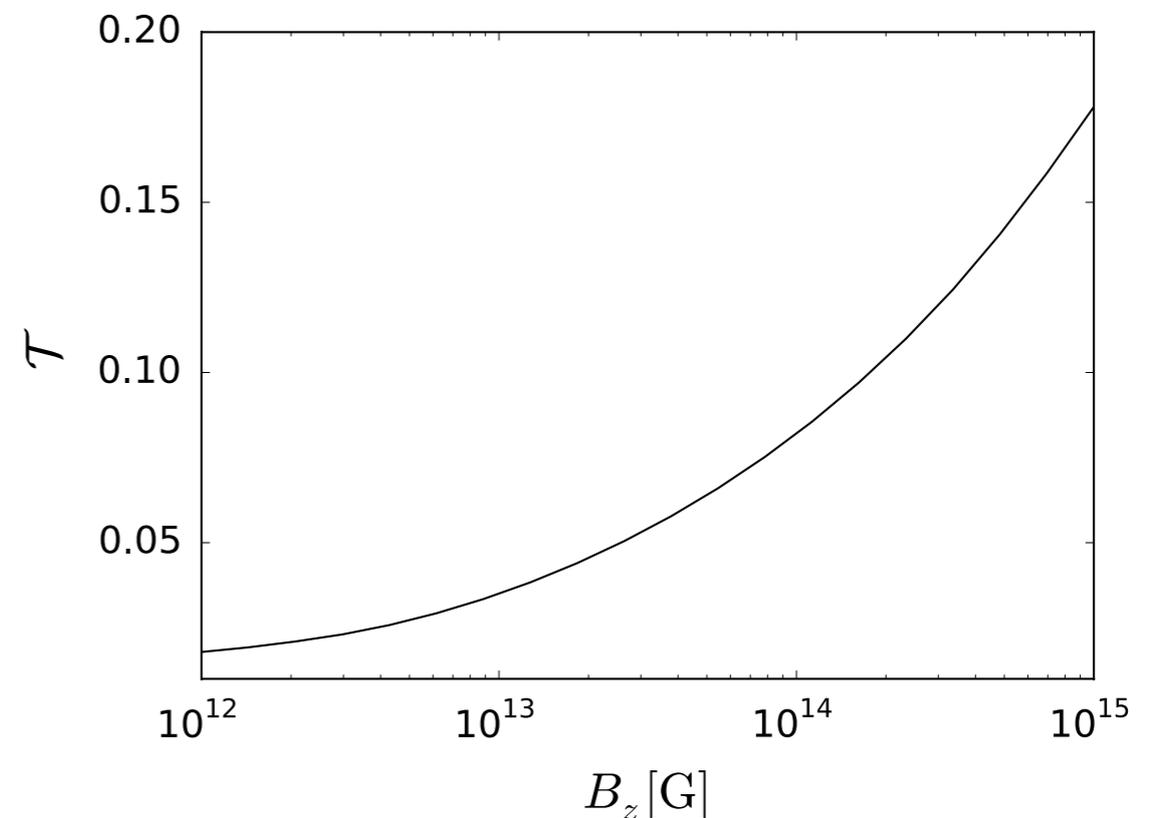
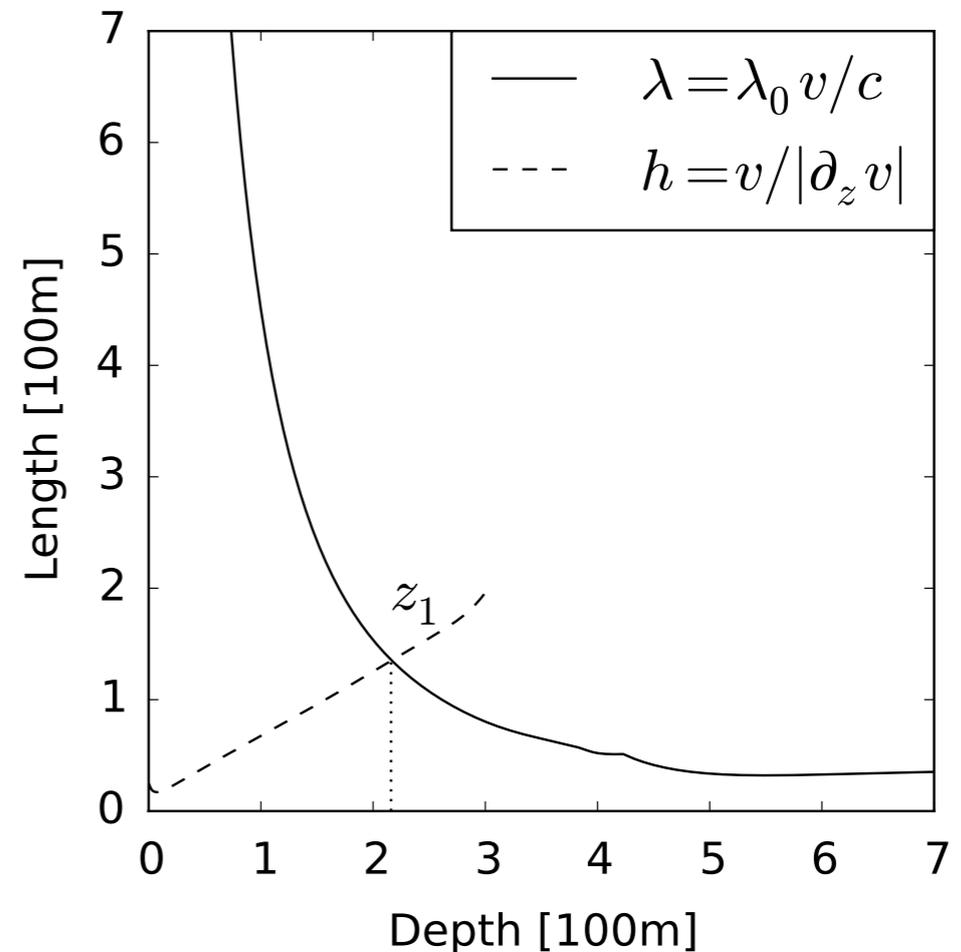
- $$\mathcal{T} \sim \frac{4Z(z_1)Z(0)}{[Z(z_1) + Z(0)]^2} \approx \frac{4v(z_1)}{c}.$$

$$Z(z) = \left[ \frac{B_z^2}{4\pi} + \mu(z) \right]^{1/2} \left[ \frac{B_z^2}{4\pi c^2} + \rho(z) \right]^{1/2}.$$

- When  $\lambda < h$ , wave propagation can be approximated by WKB with constant energy flux

- $\xi \propto v^{1/2}, \quad v \propto \rho^{-1/2}, \quad s \propto \rho^{1/4}.$

- The strain increases with density!



# Plastic heating of the crust

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- Plastic dissipation

$$\frac{dU_{\text{th}}}{dt} = -\sigma \dot{s}_{\text{pl}}, \quad |\sigma| = \sigma_{\text{cr}} + \eta |\dot{s}_{\text{pl}}|,$$

- No plastic flow if the crust is melted

$$T_{\text{melt}} \approx 2.4 \times 10^9 \rho_{12}^{1/3} \text{ K.}$$